VIBRATION AND BUCKLING ANALYSIS OF A CRACKED STEPPED COLUMN USING FINITE ELEMENT METHOD

Thesis submitted in partial fulfillment of the requirements for the degree of

Master of Technology

In

Civil Engineering

(Structural Engineering)

By

PENDRI SHASHANK REDDY

212CE2036

Department of Civil Engineering

National Institute of Technology, Rourkela

Rourkela-769008

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UNDER THE GUIDANCE OF

PROF. UTTAM KUMAR MISHRA

Department of Civil Engineering

National Institute of Technology, Rourkela

Rourkela-769008

Department of Civil Engineering
National Institute of Technology, Rourkela
Rourkela – 769 008, Odisha, India

CERTIFICATE

This is to certify that the thesis entitled, **"VIBRATION AND BUCKLING ANALYSIS OF A CRACKED STEPPED COLUMN USING FINITE ELEMENT METHOD"** submitted by **PENDRI SHASHANK REDDY,** bearing Roll No. **212CE2036** in partial fulfillment of the requirements for the award of **Master of Technology** Degree in **Civil Engineering** with specialization in **"Structural Engineering"** during 2013-14 session at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

Place: NIT Rourkela
Date: May 30, 2014

PROF. UTTAM KUMAR MISHRA
THESIS SUPERVISOR
Assistant Professor
Department of Civil Engineering
NIT Rourkela

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ABSTRACT

Components with varying cross-sections are most common in buildings and bridges as well as in machine parts. The stability of such structural members subjected to compressive forces is a topic of considerable scientific and practical interest. Tapered and stepped columns are very much useful in structural engineering because of their reduced weight compared to uniform columns for the same axial load carrying capacity or buckling load.

In the current study free vibration and buckling analysis of a cracked two stepped cantilever column is analyzed by finite element method for various compressive loads. Simple beam element with two degrees of freedom is considered for the analysis. Stiffness matrix of the intact beam element is found as per standard procedures. Stiffness matrix for cracked beam element is found from the total flexibility matrix of the cracked beam element by inverse method in line with crack mechanics and published papers by researchers. Eigen value problem is solved for free vibration analysis of the stepped column under different compressive load. Variation of free vibration frequencies for different crack depths and crack locations is studied for successive increase in compressive load. Buckling load of the column is estimated from the vibration analysis.

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NOMENCLATURE

E	Young's modulus of elasticity
I	Moment of inertia of a cross section
A	cross sectional area
F	External applied force
M	Bending moment
ω	Frequency due to vibration
u(x,t)	Longitudinal displacement
v(x,t)	Lateral displacement
M(u, x)	Bending moment
κ	Shear correction factor
a	Depth of the crack
G	Shear modulus
U^o	Strain energy of the uncracked *beam* element
U^c	Strain energy due to crack
u_i	Displacement
P_i	Force
M	Bending moment
T	Torsional moment
K	Stress intensity factor
ξ is	Relative depth

CHAPTER 1

INTRODUCTION

1. INTRODUCTION

1.1 Tapered or stepped columns

Components with varying cross-sections are most common in buildings and bridges as well as in machine parts. The stability of such structural members subjected to compressive forces is a topic of considerable scientific and practical interest that has been studied extensively, and is still receiving attention in the literature because of its relevance to structural, aeronautical and mechanical engineering. Tapered and stepped columns are very much useful practically in structural engineering because of their reduced weight compared to uniform columns for the same axial load carrying capacity or buckling load. Stepped columns are also frequently used in multistory structures where columns have to support intermediate floor loads.

To attain reduction in weight and decrease costs of steel carrying structures, the engineers tend to design steel columns as multi-stepped carriers with a non-uniform cross-section. Since columns are usually compressed by applied, self-weight, etc., one of the most important aspects of using such carriers is their elastic stability.

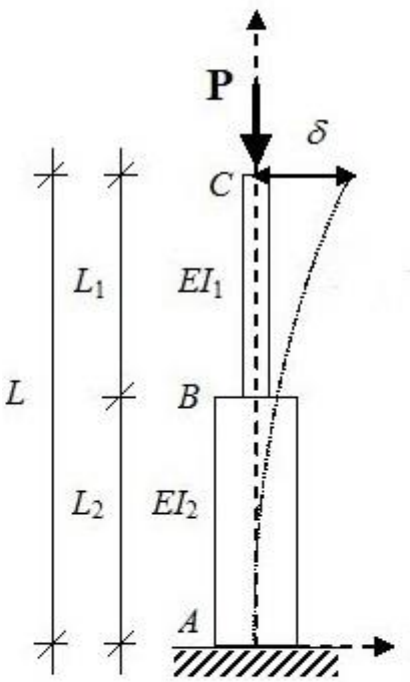


Figure1.1: stepped column with fixed-free conditions

1.2 Cracks in columns

Columns are important structural members and their stability under different cases of loading is studied by many researchers to obtain critical buckling loads and critical stresses. The cracks may develop from impact, applied cyclic load, mechanical vibrations, aero-dynamic loads etc. Due to the effect of fault or weakness that occurred due to crack in a cracked section, the stability of column may be decreased. The critical buckling loads of cracked columns are affected by effect of depths, locations, and number of cracks. Cracked section is modeled as massless rotational spring.

Since axial load and stiffness are not constant along the length of the column the analysis of a stepped column is usually much more complicated than uniform column. The change in the cross-sectional areas and distribution of loads generates discontinuity in deriving the deflection equation of a stepped beam.

Connection between foundation of a structure and super structure is most vulnerable and damage locations during and after earthquakes. So, a stepped column is used to palliate or retrofit such disadvantage. Stepped column is used to substitute rigid connections between foundation and upper structure.

The study of cracked structures and members are topic of study for decades and many researches are still going on the topic. A fault in the structure causes serious damage to the structure if left unchecked. When a structure is damaged due to crack, modal parameters assigned with the structure are greatly affected because due to damage to a structure the stiffness of the structure is decreased. Many researches are currently are concerned on damage location and damage size. The main study is concerned on extent of damage and location of damage. Many damage detecting methods use sensitivity method which uses natural frequencies. These frequency based method require a lot of computations especially for large and complex structures. Frequency changes alone are not sufficient to damage position. Similar frequency changes may occur for different damage positions. Vibration mode shapes can be heavily influenced by local damage. The greatest change occurs around the defect, thus offering the possibility of locating the damage.

The crack section is modeled as modified beam element due to presence of crack. In several studies importance is given to detection of crack that is affected by the change in natural frequencies and mode shapes of the beam.

Cracks are of different types based on their geometry, they can be classified as:

1) Transverse cracks, these cracks are perpendicular to the beam axis. These cracks cause reduction in cross-section and weakens the structure, they cause major problem to structure. A local flexibility in the stiffness of beam caused due to strain energy concentration near the crack tip is introduced. These cracks are most commonly seen in structures.
2) Longitudinal cracks, these are parallel to beam axis. When tensile load is applied perpendicular to the crack direction these cracks are a danger to structure.
3) Slant cracks, cracks formed at some angle with beam axis. Torsional behaviour of beam is affected by these cracks. These are very less effective on lateral vibrations compared to transverse cracks.
4) Breathing cracks, under tension stiffness of the structure is influenced. Non-linear vibration is caused due to breathing cracks.
5) Gaping cracks or notches, these remain open.
6) Surface cracks, these can be found by visual inspection.
7) Sub-surface cracks, they are not seen on surface. They are studied by NDT methods.

There are three modes of fracture failure, they are:

1) Mode I: opening mode, crack faces are separated in direction normal to the plane of the crack. Normal load applied on crack plane causes open crack.
2) Mode II: this is due to in plane shearing load, in this two faces of crack slides against each other or shearing mode. Stresses are developed parallel to crack direction.
3) Mode III: it is out of pale shear or tearing mode.

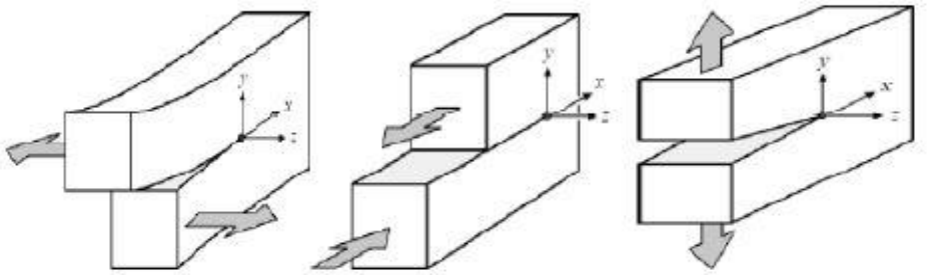


Figure1.2: Three basic modes of fracture.

In present study various end conditions are considered such as fixed-free, hinged-hinged, fixed-hinged, and fixed-fixed. The analysis depends up on the deflection function which satisfies the end conditions.

Lower bound solutions for buckling problems have been calculated by a method of successive approach done numerically by new mark method or by finite element methods. Differential equations for equilibrium of a column with small lateral displacement can be easily written in finite element equation form.

Alternate methods using wavelet analysis have been done to detect damage in structures for which the effect of small cracks may not be affected much by Eigen frequencies of the structures. Mode shapes are more sensitive to local damages when compared to natural frequencies.

CHAPTER 2

LITERATURE REVIEW

LITERATURE REVIEW

Dong C, Zhang PQ, Feng WQ, Huang TC [7] studied the relation between extent of crack and location of damage.

Springer WT, Rezniek A [27] studied the effect of modal frequencies due to change in natural frequency and there by detect the damage.

Damage in a structure is reflected in the dynamic response spectrum as a change in natural frequencies. This phenomenon has been studied by **YANG and ZHANG [35, 37]** to detect damage.

Yuen [30] has used a finite element modeling technique to establish the relationship between damage and the changes of modal parameters when a uniform cross-sectioned cantilever was subjected to damage. The damage in the cantilever beam was represented by reduction of modulus of elasticity. The first eigen parameters will reflect the location and size of the crack.

Adams and Cawley [1] have developed a method of sensitivity analysis to deduce the location of damage based on the application of a finite element method with the assumption that the modulus of elasticity in the damage area was equal to zero.

Stubbs. N [28,29] worked on damage detecting methods and evaluated a formulation that expressed change in modal stiffness in terms of modal masses, modal damping, Eigen frequencies, Eigen vector and changes.

Krawczuk and Ostachowicz [12] have developed a finite element method of an arch with transverse, one edge crack, assuming that changes the stiffness of the arch. The effects of length of crack, location of crack and changes in in-plane natural frequencies and mode shapes are found.

Rutolo and Surace [23] found a damage assessment technique using finite element method, to stimulate experimental data of cantilever beams with cracks at different locations and depths. The minimum crack depth ratio used is 0.2.

Chu.H.N, Herrmann. G and Rehfield.L.W [9] carried out a number of studies on non-linear vibrations where every problem had some particular approximations. This method is called perturbation procedure. This method can be applied to weak non-linearity, due to practical difficulties involved in calculating higher order problems. So, this method is limited to first order effects of displacements on natural frequencies.

M.M.K. Bennouna and R.G. White [17] derived a non- linear differential equation for beams with large deflections. From many experimental theoretical studies it is known that the fundamental mode shape is dependent on amplitude of vibrations, especially near clamps.

G. Falsone [9] used singularity functions to find analytical expression of deflection of an Euler-Bernoulli beam with uniform cross section, which are subjected to discontinuities in the loads, deflections and shapes.

Caddemi and Calio [4] have studied the effect of cracks on the stability of a column under different boundary conditions. They derived a 4^{th} order differential equation and followed a procedure for exact solution.

Ryu *et al*. [24] investigated the dynamic stability of cantilever Timoshenko column with a tip rigidity body and subjected to sub-tangential forces.

Vibration of beams with multiple open cracks subjected to axial force is studied by **Baric Binici, *et al*. [3]** He investigated a method to obtain the Eigen frequencies and mode shapes of beams having multiple cracks and that are subjected to axial force. The method uses one set of end conditions at crack locations. Mode shape functions of remaining parts are found. Other set of boundary conditions gives a second-order determinant that needs to be calculated for its roots. He considered both vibration and buckling load of the structure.

Ranjbaran [25] studied the stability of columns with cracks and free vibration of beams using finite element method.

Ranjbaran *et al*. [26] found a new a new and innovative method for longitudinal dynamic characteristics of beams with multiple cracks. Numerical solutions are obtained from finite element method.

Wang *et al*. [6] analyzed buckling of weak columns with cracks at an internal locations and cracked is modelled as massless spring.

M. Arif Gruel. M and Kisa [21] analyzed the buckling of slender prismatic columns with single edge crack subjected to concentric vertical loads using transfer matrix method.

M. Arif Gruel [22] studied the buckling of slender prismatic columns that are weakened by multiple edge cracks.

Liebowitz [15, 16] *et al.* analyzed the behaviour of notched columns under the eccentric loads.

Luban [17] worked on experimental determination of energy release rates, for columns that are notched and subjected to tension that provides a derivation of stress intensity factors.

Chondros. T. G [5] *et al.* developed a continuous cracked beam theory of lateral vibration of single edge or double edge open cracked Euler-Bernoulli beams using Hu-Washizu bar variational formulation to investigate the differential equation and boundary conditions of cracked beam as one- dimensional continuum. The displacement field about the crack was used to modify the stress and displacement field throughout the bar. A steel beam with a double-edge crack was taken as an example and results compared well with previous experimental data. They extended their research for a beam with breathing crack.

Shifrin. E. I, *et al*. [8] investigated a technique for finding natural frequencies of a vibrating beam having arbitrary number of transverse open cracks. The main advantage of this method is decrease in the dimension of the matrix involved in the calculation, so computation time required for evaluating natural frequencies compared to alternative methods is reduced.

Buckling of multi-step crack columns which are subjected shear deformation is analyzed by **Li. Q. S, *et al.*** [13, 14]. The differential equation for buckling of one-step cracked column with shear deformation is established and its solution is found. A new approach that combines the transfer matrix method (TMM) and the exact buckling solution of a one-step column and is given for analyzing the entire and partial buckling of a multistep column with different end conditions, with or without cracks and shear deformation, subjected to concentrated axial load. The main advantage of this method is that the eigen-value equation for buckling of a multi-step column with an arbitrary number of cracks, any kind of two end supports and various spring supports at intermediate points can be determined from a system of two linear equations.

Lee *et al*. [18] later extended the method and proposed classes of exact solutions for buckling of multi-step non-uniform columns with an arbitrary number of cracks subjected to concentrated and distributed axial loads.

Abraham and Brandon and Brandon and Abraham [3, 7] presented a method utilizing substructure normal modes to predict the vibration properties of a cantilever beam with a closing crack. The full eigen- solution of a structure containing substructures each having large numbers of degrees of freedom can be costly in computing time.

CHAPTER 3

THEORETICAL ANALYSIS AND FORMULATION

THEORY AND FORMULATION

Considering an un-cracked Timoshenko beam of area of cross section *A*, moment of inertia *I*, mass density *ρ*, the coupled equations for free vibrations of beam are given by Timoshenko (1967).

$$
\begin{aligned}
\rho A \frac{\partial^2 q_1(x,t)}{\partial t^2} &= \kappa G A\left(\frac{\partial^2 q_1(x,t)}{\partial x^2} - \frac{\partial q_2(x,t)}{\partial x}\right) \\
\rho I \frac{\partial^2 q_2(x,t)}{\partial t^2} &= \kappa G A\left(\frac{\partial q_1(x,t)}{\partial x} - q_2(x,t)\right) + EI\frac{\partial^2 q_2(x,t)}{\partial x^2}
\end{aligned} \tag{1}
$$

Where, q_1 and q_2 represent the transverse deflection and rotation of the beam respectively. *E* and *G* are the Young's modulus and Shear modulus respectively and *κ* is the shear correction factor. The general governing equation is derived by substituting the beam's kinematic and potential energies in Lagrange's equation:

$$
\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} = 0 \tag{2}
$$

The Kinetic energy:

$$
T = \frac{1}{2}\int_0^L \rho A\left(\frac{\partial q_1(x,t)}{\partial t}\right)^2 dx + \frac{1}{2}\int_0^L J\left(\frac{\partial q_2(x,t)}{\partial t}\right)^2 dx \tag{3}
$$

The potential energy U of the beam due to elastic bending with shear deformation and axial in plane force is: $U=U_1+U_2$

Where,

U_1 = Strain energy associated due to bending with transverse shear

U_2 = Work done by the initial in-plane stresses and the nonlinear strain

The expression of strain energy U becomes

$$
U = \frac{1}{2}\int_0^L EI\left(\frac{\partial q_2(x,t)}{\partial x}\right)^2 dx + \frac{1}{2}\int_0^L \kappa G A\left(\frac{\partial q_1(x,t)}{\partial x} - q_2\right)^2 dx + \frac{1}{2}\int_0^L P\left(\frac{\partial q_1(x,t)}{\partial x}\right)^2 dx \tag{4}
$$

The additional strain energy due to the existence of crack is considered according to Zheng and Kessissoglou (2004) and the bending stiffness is modified.

The energies for the beam with cracks can be written in matrix form as

$$U_1 = \frac{1}{2}\{q\}^T[K_e]\{q\} \tag{5}$$

$$U_2 = \frac{1}{2}\{q\}^T[K_g]\{q\} \tag{6}$$

$$T = \frac{1}{2}\{\dot{q}\}^T[M]\{\dot{q}\} \tag{7}$$

Where, $[K_e]$ = Bending stiffness matrix with shear deformation of the beam

$\lfloor K_g \rfloor$ = Geometric stiffness or stress stiffness matrix of the beam

$[M]$ = Consistent mass matrix of the beam

Substituting in the Lagrange's equation and on simplification the equation of motion for vibration of cracked beam subjected to in plane load P reduces to matrix form as:

$$[M]\{\ddot{q}\} + [[K_e] - P[K_g]]\{q\} = 0 \tag{8}$$

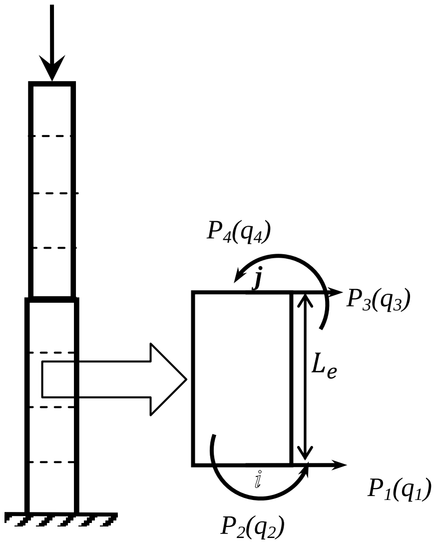
'q' is the vector of degrees of freedoms. $[M]$, $[K_e]$ and $\lfloor K_g \rfloor$ are the mass, elastic stiffness and geometric stiffness matrix of the beam respectively.

The Eq. 8 leads to the solution to the free vibration analysis of the beam problem, which is given by

$$\lfloor [K] - \omega^2[M] \rfloor \{q\} = 0 \tag{9}$$

Where $[K] = [K_e] – P[K_g]$

3.1 Finite Element Analysis



$q_1 = deflection\ at\ node\ i$

$q_2 = slope\ at\ node\ i$

$q_3 = deflection\ at\ node\ j$

$q_4 = slope\ at\ node\ j$

$L_e = length\ of\ the\ element$

Figure 3.1: Intact Stepped Column discretized into beam elements with 2 dof per node for finite element analysis

The displacement model is taken as a polynomial given by,

$$q = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

Shape functions as derived by Cook *et al*. (2003) are given by,

$$\begin{aligned} N_1 &= 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3} \\ N_2 &= x - \frac{2x^2}{l} + \frac{x^3}{l^2} \\ N_3 &= \frac{3x^2}{l^2} - \frac{2x^3}{l^3} \\ N_4 &= -\frac{x^2}{l} + \frac{x^3}{l^2} \end{aligned} \tag{10}$$

Where

$$[N] = [N_1 \quad N_2 \quad N_3 \quad N_4]$$

$[N]$ = Shape function matrix

Similarly, the strain displacement matrix coefficients in line with Cook *et al*. (2003) is given by

$$\begin{aligned}
B_1 &= -\frac{6}{l^2} + \frac{12x}{l^3} \\
B_2 &= -\frac{4}{l} + \frac{6x}{l^2} \\
B_3 &= \frac{6}{l^2} - \frac{12x}{l^3} \\
B_4 &= -\frac{2}{l} + \frac{6x}{l^2}
\end{aligned} \tag{11}$$

$$\therefore [B] = [B_1 \quad B_2 \quad B_3 \quad B_4]$$

Where $[B]$ = strain displacement matrix

3.2 Element Stiffness matrix

The stiffness matrix for 2 degree of freedom *(v,θ)* for bending in the *xy*-plane for a two-noded Timoshenko beam finite element with shear deformation in line with Gounaris and Papazoglou (1992) is given by,

$$[K_e] = \frac{EI}{L(L^2 + 12\beta)} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 + 12\beta & -6L & 2L^2 - 12\beta \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 - 12\beta & -6L & 4L^2 + 12\beta \end{bmatrix} \tag{12}$$

Where, L = length of the element

E = young's modulus of elasticity

I = moment of inertia of the section with respect to *z*-axis,

and

$$\beta = \frac{EI}{\kappa GA}$$

where, κ = shear correction factor

G = the shear modulus

A = area of the cross-section of the element

3.3 Element Mass matrix

$$[M] = \int_0^L [N]^T [\rho A][N]dx \tag{20}$$

$$[M] = \frac{\rho A l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \tag{21}$$

Where, ρ = Mass density of the beam material

A= Cross-sectional area of the beam element

3.4 Element Geometric Stiffness Matrix

$$[K_g] = \int_0^l \left[\frac{\partial N}{\partial x}\right]^T \left[\frac{\partial N}{\partial x}\right] dx$$

$$[K_g] = \frac{1}{30L}\begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 3L^2 \end{bmatrix} \tag{13}$$

Where A= cross-sectional area of the element

ρ = Mass density of the beam

3.5 STIFFNESS MATRIX FOR A CRACKED BEAM ELEMENT

The key problem in using FEM is how to appropriately obtain the stiffness matrix for the cracked beam element. The most convenient method is to obtain the total flexibility matrix first and then take inverse of it. The total flexibility matrix of the cracked beam element includes two parts. The first part is original flexibility matrix of the intact beam. The second part is the additional flexibility matrix due to the existence of the crack, which leads to energy release and additional deformation of the structure.

3.6 Elements of the overall additional flexibility matrix C_{ovl}

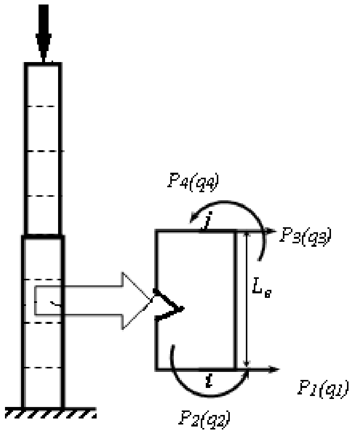


Figure 3.2 Cracked beam element with 2 degree of freedom

The fig. 3.3 shows a typical cracked beam element with a rectangular cross section. The left hand side end node ***i*** is assumed to be fixed, while the right hand side end node ***j*** is subjected to shearing force P_1 and bending moment P_2. The corresponding generalized displacements are denoted as q_1 and q_2.

B = Breath of the beam

h = Depth of the beam

a = crack depth

L_c = Distance between the right hand side end node j and the crack location

Le = Length of the beam element

A = Cross-sectional area of the beam

I = Moment of inertia

According to Dimarogonas *et al.* (1983) and Tada *et al.* (2000) the additional strain energy due to existence of crack can be expressed as

$$\Pi_C = \int_A G dA_C \tag{14}$$

Where, G = the strain energy release rate and

A_C = the effective cracked area.

$$G=\frac{1}{E'}\left[\left(\sum_{n=1}^{2}K_{In}\right)^2+\left(\sum_{n=1}^{2}K_{IIn}\right)^2+k\left(\sum_{n=1}^{2}K_{IIIn}\right)^2\right] \tag{15}$$

Where, $E' = E$ for plane stress

$= E/1-\upsilon^2$ for plane strain

$k = 1+\upsilon$

K_I, K_{II} *and* K_{III} = stress intensity factors for opening, sliding and tearing type cracks respectively

Neglecting effect of axial force and for open cracks, Eq.15 can be written as

$$G=\frac{I}{E'}\left[(K_{I1}+K_{I2})^2+K_{II1}^2\right] \tag{16}$$

The expressions for stress intensity factors from earlier studies are given by,

$$\begin{aligned} K_{I1} &= \frac{6P_1L_c}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right)\\ K_{I2} &= \frac{6P_2}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right)\\ K_{II1} &= \frac{P_2}{bh}\sqrt{\pi\xi}F_{II}\left(\frac{\xi}{h}\right) \end{aligned} \tag{17}$$

Where,

$$F_I(s)=\sqrt{\frac{\tan\left(\pi s/2\right)}{\left(\pi s/2\right)}}\left[\frac{0.923+0.199\left(1-\sin\left(\pi s/2\right)\right)^4}{\cos\left(\pi s/2\right)}\right]$$

$$F_{II}(s)=\frac{1.122-0.561s+0.085s^2+0.180s^3}{\sqrt{1-s}}$$

Here, $s=\frac{\xi}{h}$ (crack depth during the process of penetrating from zero to final depth), and

$F_1(s)$ *and* $F_{II}(s)$ are the correction factors for stress intensity factors

From definition, the elements of the overall additional flexibility matrix C_{ij} can be expressed as

$$C_{ij} = \frac{\partial \delta_i}{\partial P_j} = \frac{\partial^2 \prod_c}{\partial P_i \partial P_j} \quad (i,j = 1, 2,) \tag{18}$$

Substituting Eq. 27 in Eq. 26 and subsequently in Eq. 24 we get,

$$C_{ij} = \frac{b}{E'} \frac{\partial^2}{\partial P_i \partial P_j} \int \begin{bmatrix} \left\{ \frac{6P_1 L_c}{bh^2} \sqrt{\pi\xi} F_1\left(\frac{\xi}{h}\right) + \frac{6P_2}{bh^2} \sqrt{\pi\xi} F_1\left(\frac{\xi}{h}\right) \right\}^2 + \\ \left\{ \frac{P_1}{bh} \sqrt{\pi\xi} F_{II}\left(\frac{\xi}{h}\right) \right\}^2 \end{bmatrix} d\xi \tag{19}$$

Substituting *i*, *j (1, 2)* values, we get

$$C_{11} = \frac{2\pi}{E'b} \left[\frac{36 L_c^2}{h^2} \int_0^{\frac{a}{h}} x F_1^2(x) dx + \int_0^{\frac{a}{h}} x F_{II}^2(x) dx \right] \tag{20}$$

$$C_{12} = \frac{72\pi L_c}{E'bh^2} \int_0^{\frac{a}{h}} x F_1^2(x) dx = C_{21} \tag{21}$$

$$C_{22} = \frac{72\pi}{E'bh^2} \int_0^{\frac{a}{h}} x F_1^2(x) dx \tag{22}$$

Now, the overall flexibility matrix C_{ovl} is given by,

$$[C_{ovl}] = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \tag{23}$$

3.6.1 Flexibility matrix C_{intact} of the intact beam element

$$[C_{intact}] = \begin{bmatrix} \frac{L_e^3}{3EI} & \frac{L_e^2}{2EI} \\ \frac{L_e^2}{2EI} & \frac{L_e}{EI} \end{bmatrix} \tag{24}$$

3.6.2 Total flexibility matrix C_{tot} of the cracked beam element

$$[C_{total}] = [C_{intact}] + [C_{ovl}]$$

$$[C_{total}] = \begin{bmatrix} \frac{L_e^3}{3EI} + C_{11} & \frac{L_e^2}{2EI} + C_{12} \\ \frac{L_e^2}{2EI} + C_{21} & \frac{L_e}{EI} + C_{22} \end{bmatrix} \tag{25}$$

3.6.3 Stiffness matrix K_C of a cracked beam element:

From the equilibrium conditions, the stiffness matrix K_c of a cracked beam element can be obtained as

$$[K_{crack}] = \{L\}[C_{tot}^{-1}]\{L\}^T \tag{26}$$

Where L is the transformation matrix for equilibrium condition

$$\{L\} = \begin{Bmatrix} -1 & 0 \\ -L_e & -1 \\ 1 & 0 \\ 0 & 1 \end{Bmatrix} \tag{27}$$

The results are presented for vibration of beams with cracks using the present formulation. The boundary conditions are

- Fixed end: $v = 0$, and $\theta = 0$
- Free end: no restraint

CHAPTER 4

COMPUTATIONAL PROCEDURE

COMPUTATIONAL PROCEDURE

To obtain the frequencies for vibration of beam for different crack locations, crack depths and different percentages of loading by using finite element method, a finite element computational software Matlab R12b is used.

Matlab R12b is user friendly, its name is derived from its functioning formula translating system, and it is used for optimization of mathematical process. Matlab R12b is used in computationally intensive areas such as numerical weather prediction, finite element analysis, computational fluid dynamics, computational physics and computational chemistry.

4.1 Current study

Problem considered under current study is a 2 stepped column with each step 0.15m and total height of column 0.3m with depth of cross section of upper step is 10mm and depth of cross section of below is 10mm and the breadth is maintained a constant value of 10mm. the young's modulus of the material is 68.215GPa, poisons ratio is 0.28, and the density of the material is 2569 kg/m^3.

In the current study free vibration and buckling analysis of a cracked two stepped cantilever column is analyzed by finite element method for various compressive load. Simple beam element with two degrees of freedom is considered for the analysis. Stiffness matrix of the intact beam element is found, Stiffness matrix for cracked beam element is found from the total flexibility matrix of the cracked beam element by inverse method in line with crack mechanics.

Eigen value problem is solved for free vibration analysis of the stepped column under different compressive load. Variation of free vibration frequencies for different crack depths and crack locations is studied for successive increase in compressive load. Buckling load of the column is estimated from the vibration analysis.

First critical buckling load of the stepped column is obtained from the computational procedure by using Matlab R12b. Then frequencies of the stepped column by varying different parameters such as crack depth, crack location, loading are found.

CHAPTER 5

RESULTS AND DISCUSSION

RESULTS AND DISCUSSION

5.1 Convergence study

To study the correct approximation of results obtained the convergence study is done, for this each element is sub-divided to some number of elements and with certain load vibration analysis is done. And the results for different number of elements are compared and seen that those values are approximately equal.

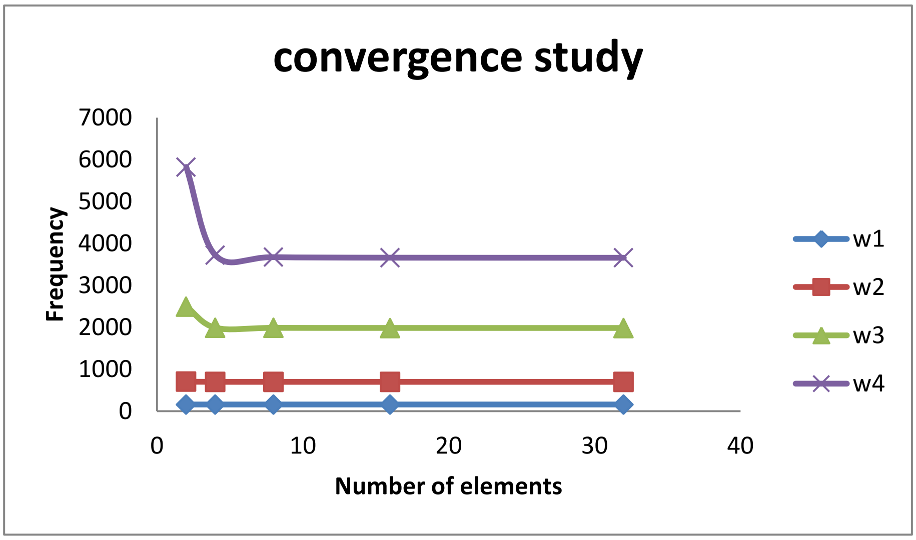


Figure 5.1: A graph between number of elements and frequency for different frequencies

Different number of elements for each section 2, 4,8,16 are considered and for fixed-free end condition, their corresponding frequencies for free vibration are found and compared with a graph. It is observed that frequencies converge for different number of elements.

5.2 Validation of the results of current study

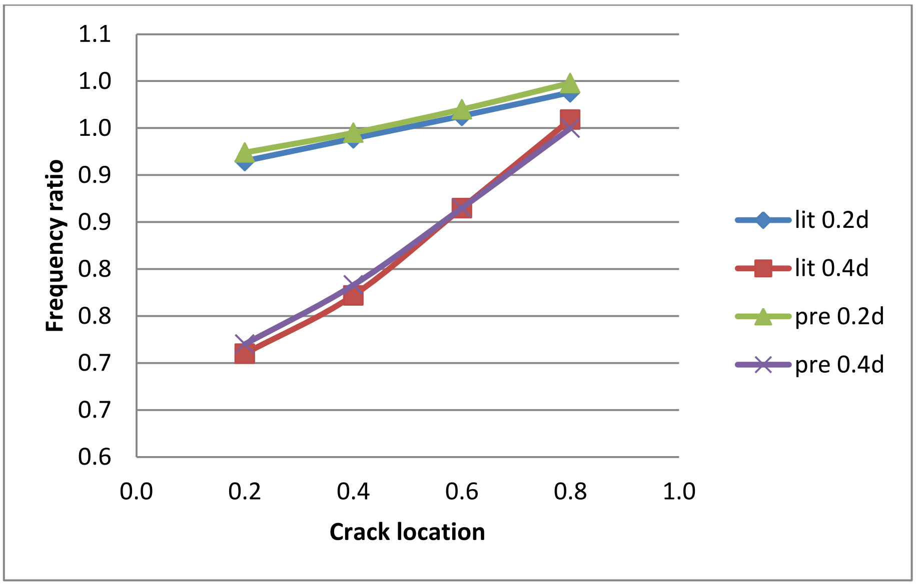


Figure 5.2: A graph between frequency ratio and crack location for validation of result.

This is result of variation of frequency ratio with crack location from Talaat H. Abdel-Lateef, Magdy Israel Salama, Buckling of slender prismatic columns with multiple edge cracks using energy approach, Alexandria Engineering Journal(2013) 52, 741-747.

A clamped- free ended column shown in with the following data is considered.
h = b= 20 cm, L =3m, a =0.3 h = 6 cm and xc = 0.3 L =0.90 m

5.3 Observations for Variation of Frequency Ratio of Frequency with Change in Crack Location for Different Crack Depths

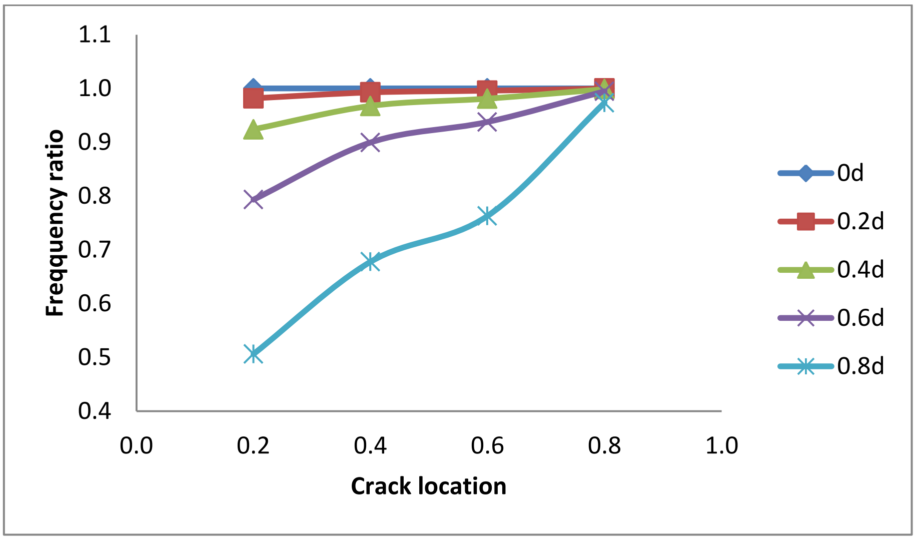


Figure 5.3: Variation of frequency ratio of 1st frequency with change in crack location for different crack depths under free vibration.

For 1st frequency and free vibration condition with no load and crack depth 0.2d the change in frequency ratio is minimal. For 0.4d the change from 0.2L to 0.4L is considerable but after that the change is negligible. For 0.6d the frequency changes almost linearly. And for 0.8d the frequency ratio increases linearly from 0.2L to 0.8L.

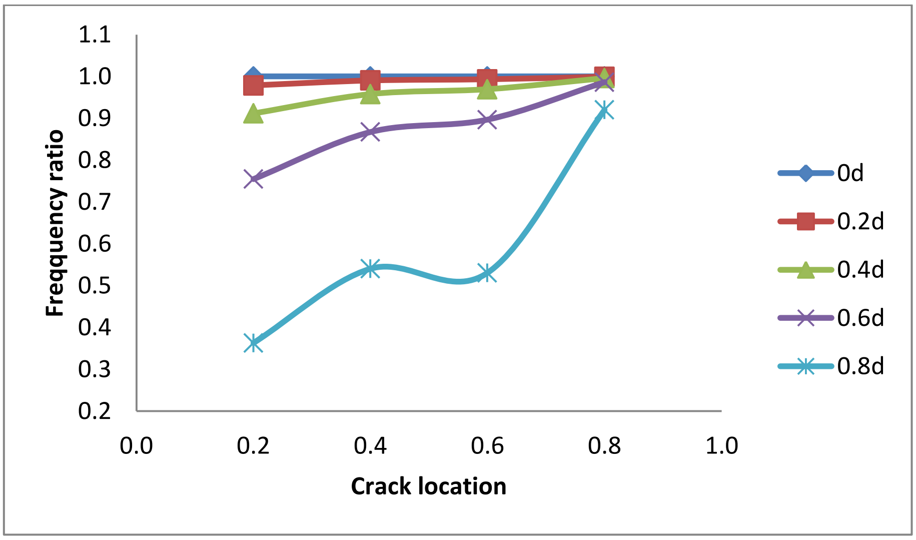


Figure 5.4: Variation of frequency ratio of 1st frequency with change in crack location for different crack depths under free vibration with a compressive load 0.2P.

For 1st frequency and with free vibration under 0.2P compressive load the change in frequency ratio for 0.2d the change with respect to different crack locations is minimal. For crack depth 0.4d the change is small and linear. The change is considerable and linear for crack depth 0f 0.6d and for crack depth of 0.8d the variation of frequency ratio with crack location is it increases from 0.2L to 0.4L and the decreases at 0.6L and increases at 0.8L.

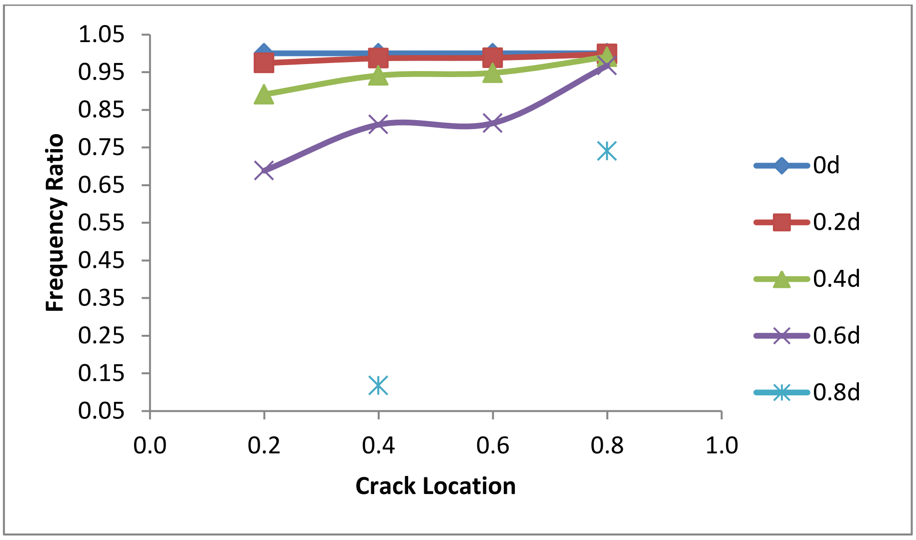


Figure 5.5: Variation of frequency ratio of 1st frequency with change in crack location for different crack depths under free vibration with a compressive load 0.4P.

For free vibration analysis under 0.4P the frequency ratio of 1st frequency for 0.2d the change is negligible for different crack locations. For 0.4d it changes linearly. For 0.6d the change is considerable and linear. It is less at 0.2L and increase to 1 at 0.8L. For 0.8d crack depth column buckles for locations 0.2L and 0.6L.

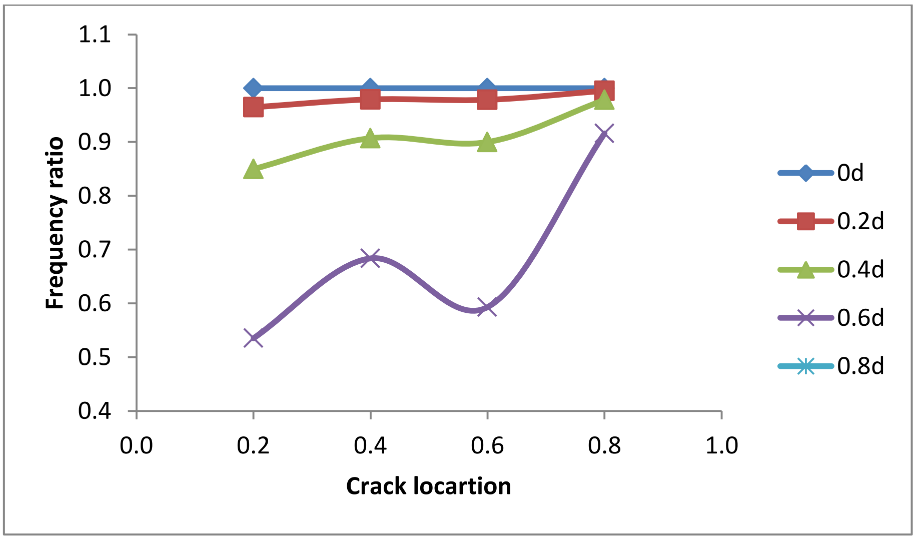


Figure 5.6: Variation of frequency ratio of 1st frequency with change in crack location for different crack depths under free vibration with a compressive load 0.6P.

For 1st frequency and with free vibration under 0.2P compressive load the change in frequency ratio for 0.2d the change with respect to different crack locations is minimal. For crack depth 0.4d the change is small and linear. And for crack depth of 0.6d the variation of frequency ratio with crack location is it increases from 0.2L to 0.4L and the decreases at 0.6L and increases at 0.8L. At crack depth 0.8d the column buckles. Frequency decreases at 0.6L because the location falls in second step for which area of cross section of that step is less than the first step and under higher compressive load and effected to greater crack depth, it again increases near to free end.

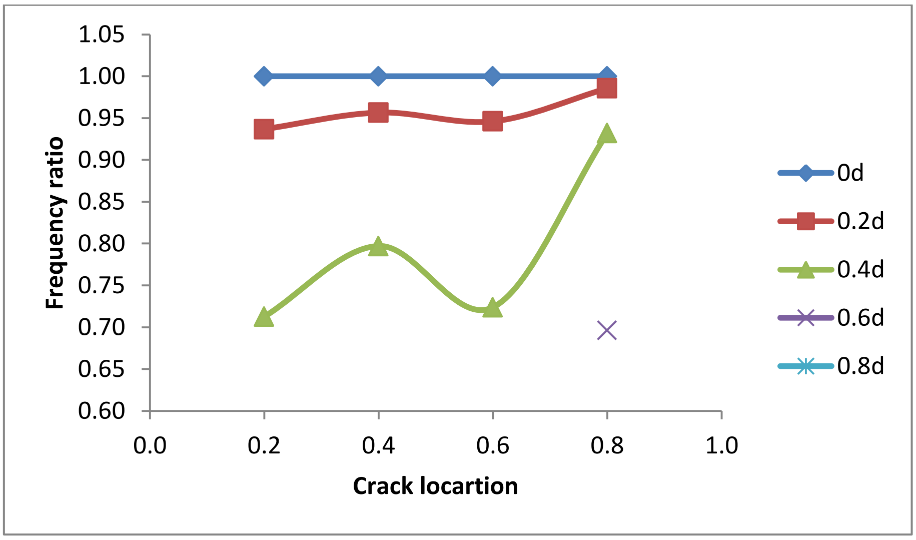


Figure 5.7: Variation of frequency ratio of 1st frequency with change in crack location for different crack depths under free vibration with a compressive load 0.8P.

The change in frequency ratio for different crack locations for varying crack depths the change of frequency ration for 0.2d crack is linear. For 0.4d the change is non-linear and for crack depth 0.6d column buckles for crack at all locations except for 0.8L. For crack near depth 0.8d the column buckles. Frequency decreases at 0.6L because the location falls in second step for which area of cross section of that step is less than the first step and under higher compressive load and effected to greater crack depth, it again increases near to free end.

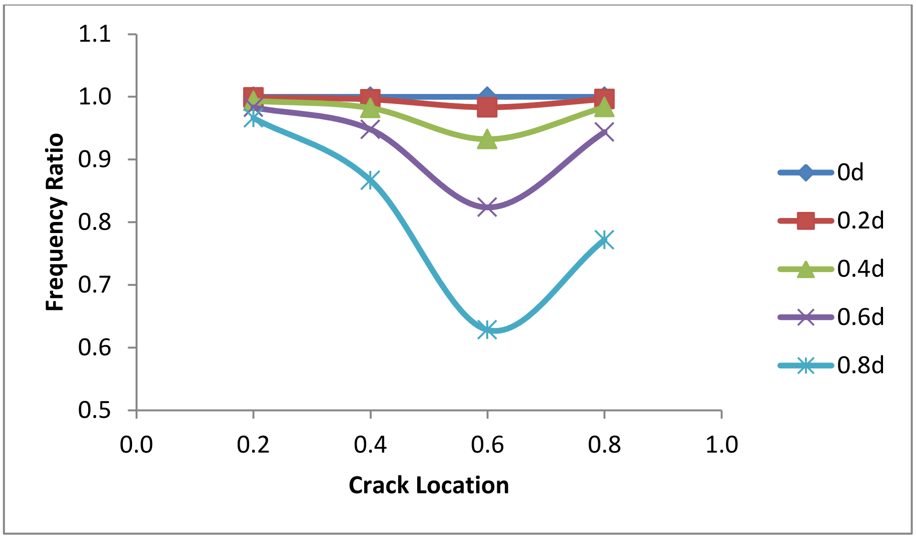


Figure 5.8: Variation of frequency ratio of 2nd frequency with change in crack location for different crack depths under free vibration.

For 2nd frequency and with free vibration under no load the change in frequency ratio for 0.2d the change with respect to different crack locations is minimal but at 0.6L it slightly decreases. For crack depth 0.4d the change is considerable at 0.6L. The change is decrease in ratio for crack depth 0f 0.6d and for crack depth of 0.8d the variation of frequency ratio linear but at 0.6L there is a big decrease in frequency ratio. Frequency decreases at 0.6L because the location falls in second step for which area of cross section of that step is less than the first step and it again increases near to free end.

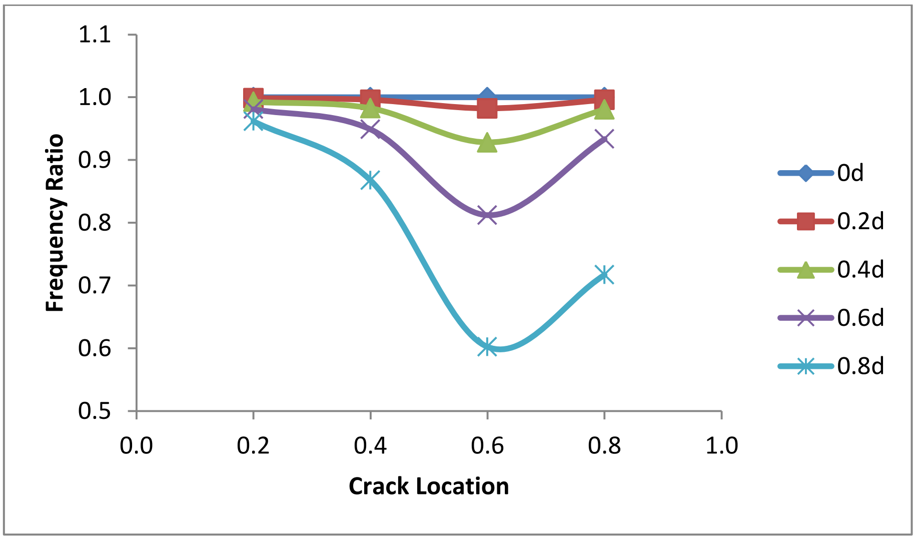


Figure 5.9: Variation of frequency ratio of 2nd frequency with change in crack location for different crack depths under free vibration with a compressive load 0.2P.

For 2nd frequency and with free vibration under 0.2P load the change in frequency ratio for 0.2d the change with respect to different crack locations is minimal but at 0.6L it slightly decreases. For crack depth 0.4d the change is considerable at 0.6L. The change is decrease in ratio for crack depth 0f 0.6d and for crack depth of 0.8d the variation of frequency ratio linear but at 0.6L there is a big decrease in frequency ratio. Frequency decreases at 0.6L because the location falls in second step for which area of cross section of that step is less than the first step and it again increases near to free end.

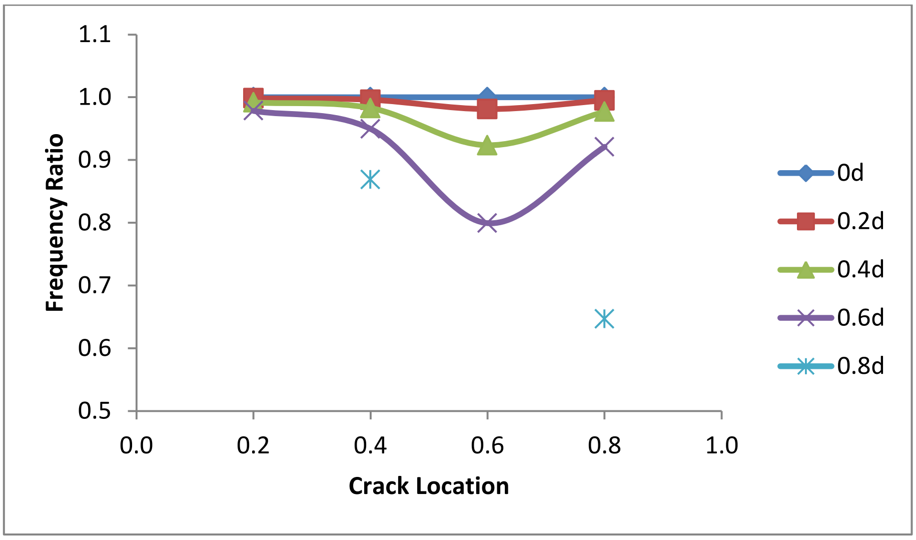


Figure 5.10: Variation of frequency ratio of 2nd frequency with change in crack location for different crack depths under free vibration with a compressive load 0.4P.

For 2nd frequency and with free vibration under 0.4P load the change in frequency ratio for 0.2d the change with respect to different crack locations is minimal but at 0.6L it slightly decreases. For crack depth 0.4d the change is considerable at 0.6L. The change is decrease in ratio for crack depth 0f 0.6d and for crack depth of 0.8d the column buckles except for 0.8L. Frequency decreases at 0.6L because the location falls in second step for which area of cross section of that step is less than the first step and it again increases near to free end.

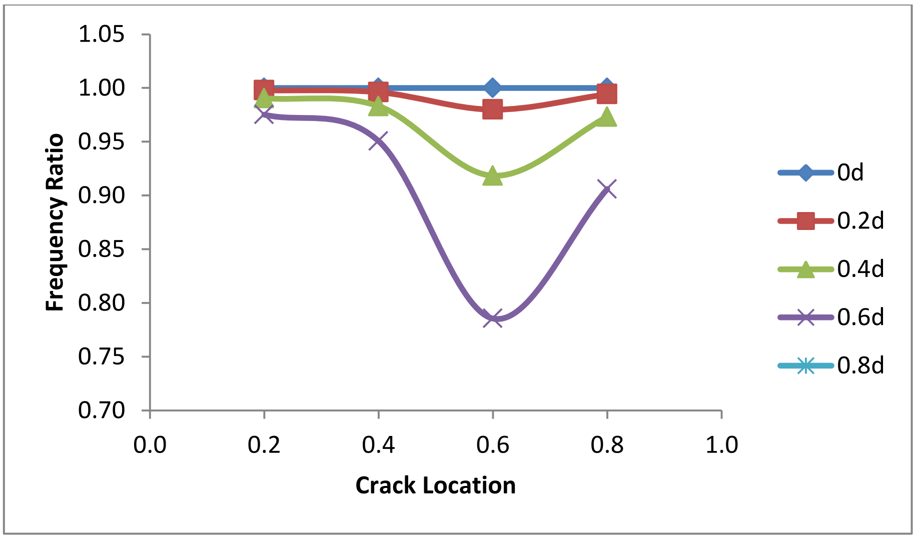


Figure 5.11: Variation of frequency ratio of 2nd frequency with change in crack location for different crack depths under free vibration with a compressive load 0.6P.

For 2nd frequency and with free vibration under 0.6P load the change in frequency ratio for 0.2d the change with respect to different crack locations is minimal but at 0.6L it slightly decreases. For crack depth 0.4d the change is considerable at 0.6L. The change is decrease in ratio for crack depth 0f 0.6d and for crack depth of 0.8d the column buckles. Frequency decreases at 0.6L because the location falls in second step for which area of cross section of that step is less than the first step and under higher compressive load and effected to greater crack depth, it again increases near to free end.

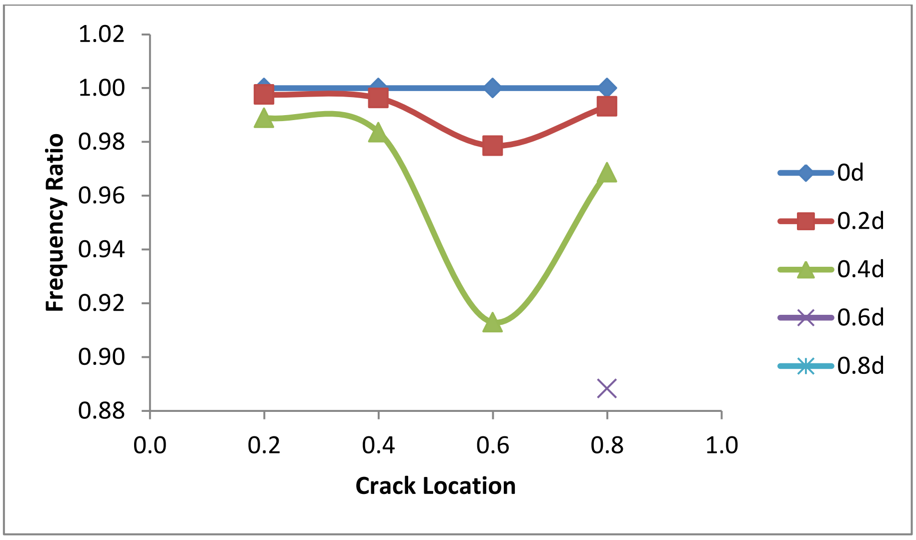


Figure 5.12: Variation of frequency ratio of 2nd frequency with change in crack location for different crack depths under free vibration with a compressive load 0.8P.

For 2nd frequency and with free vibration under 0.8P load the change in frequency ratio for 0.2d the change with respect to different crack locations is minimal but at 0.6L it decreases. For crack depth 0.4d it decreases considerably at 0.6L. For crack depth 0f 0.6d column buckles except for crack location 0.8Land for crack depth of 0.8d the column buckles. Frequency decreases at 0.6L because the location falls in second step for which area of cross section of that step is less than the first step and under higher compressive load and effected to greater crack depth, it again increases near to free end.

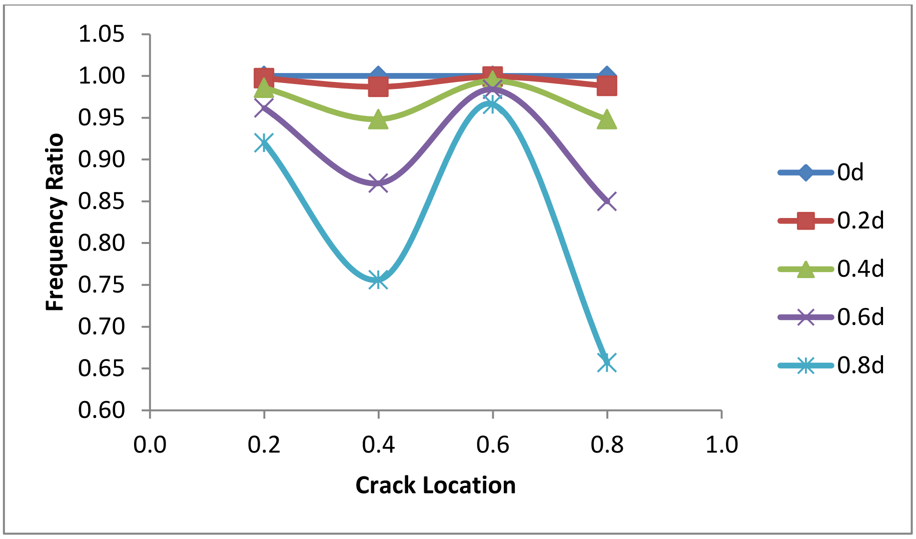


Figure 5.13: Variation of frequency ratio of 3rd frequency with change in crack location for different crack depths under free vibration.

For a stepped beam under free vibration and a no load applied case the decrease in frequency ratio for 0.2d is almost negligible. For 0.4d frequency ratio decrease slightly at 0.4L and again reaches 1 at 0.8L frequency ratio again decreases. For 0.6d the frequency ratio decreases considerably at 0.4L and reaches 1 at 0.6L and again decreases at 0.8Ld. For crack depth 0.8d it follows same trend as 0.6d and 0.4d curve but here the decrease is higher.

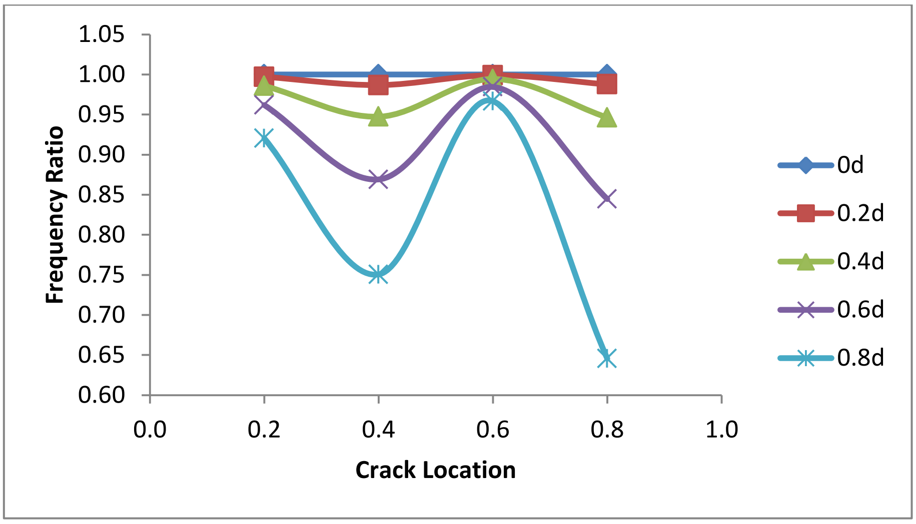


Figure 5.14: Variation of frequency ratio of 3rd frequency with change in crack location for different crack depths under free vibration with a compressive load 0.2P.

For a stepped beam under free vibration and load applied 0.2P case the decrease in frequency ratio for 0.2d is almost negligible. For 0.4d frequency ratio decrease slightly at 0.4L and again reaches 1 at 0.8L frequency ratio again decreases. For 0.6d the frequency ratio decreases considerably at 0.4L and reaches 1 at 0.6L and again decreases at 0.8L. for crack depth 0.8d it follows same trend as 0.6d and 0.4d curve but here the decrease is higher.

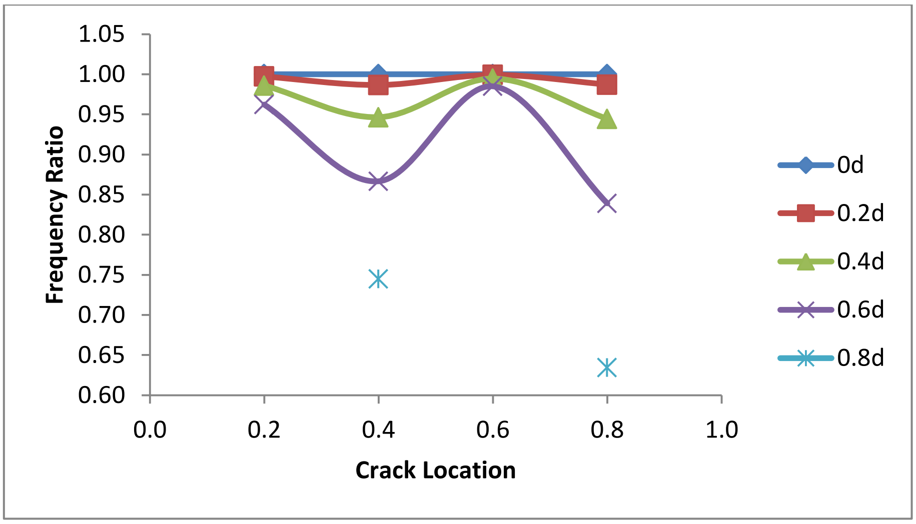


Figure 5.15: Variation of frequency ratio of 3rd frequency with change in crack location for different crack depths under free vibration with a compressive load 0.4P.

For a stepped beam under free vibration and load applied 0.2P case the decrease in frequency ratio for 0.2d is almost negligible. For 0.4d frequency ratio decrease slightly at 0.4L and again reaches 1 at 0.8L frequency ratio again decreases. For 0.6d the frequency ratio decreases considerably at 0.4L and reaches 1 at 0.6L and again decreases at 0.Ld. for crack depth 0.8d it buckles at 0.2L and 0.8L.

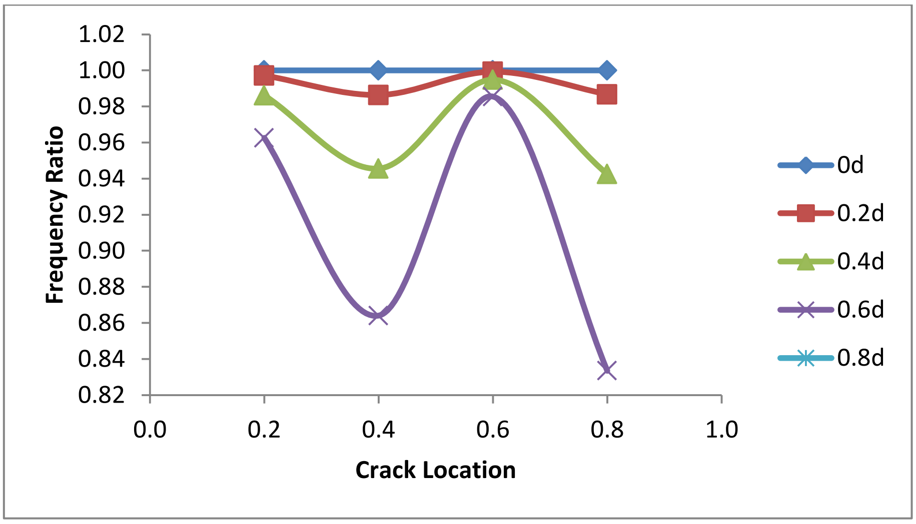


Figure 5.16: Variation of frequency ratio of 3rd frequency with change in crack location for different crack depths under free vibration with a compressive load 0.6P.

For a stepped beam under free vibration and load applied 0.2P case the decrease in frequency ratio for 0.2d is almost negligible. For 0.4d frequency ratio decrease slightly at 0.4L and again reaches 1 at 0.8L frequency ratio again decreases. For 0.6d the frequency ratio decreases considerably at 0.4L and reaches 1 at 0.6L and again decreases at 0.8L. for crack depth 0.8d it buckles for all locations.

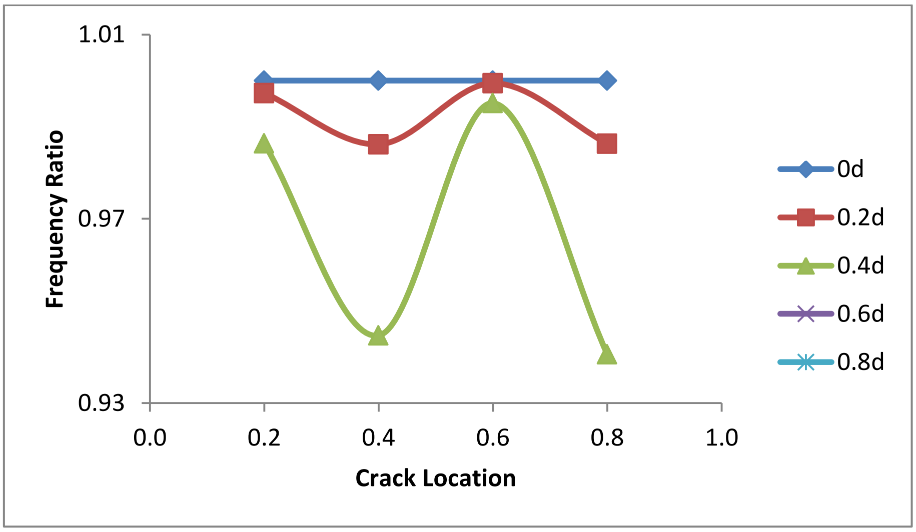


Figure 5.17: Variation of frequency ratio of 3rd frequency with change in crack location for different crack depths under free vibration with a compressive load 0.8P.

For a stepped beam under free vibration and load applied 0.2P case the decrease in frequency ratio for 0.2d is almost negligible. For 0.4d frequency ratio decrease slightly at 0.4L and again reaches 1 at 0.8L frequency ratio again decreases. For 0.6d the column buckles except at 0.8L. For crack depth 0.8d it buckles for all locations.

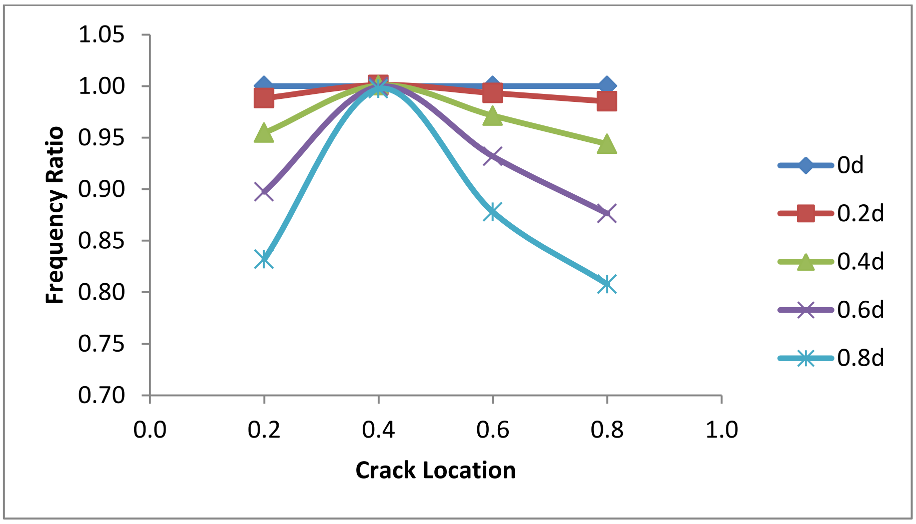


Figure 5.18: Variation of frequency ratio of 4th frequency with change in crack location for different crack depths under free vibration.

For free vibration analysis of stepped beam under no load condition the frequency ratio of 4th frequency ratio is constant for 0.2d and for 0.4d the changes are slight, for 0.6d increases from 0.2L to 0.4L reaches 1 and then decrease linearly up to 0.8L and for 0.8d the changes are similar to 0.6d.

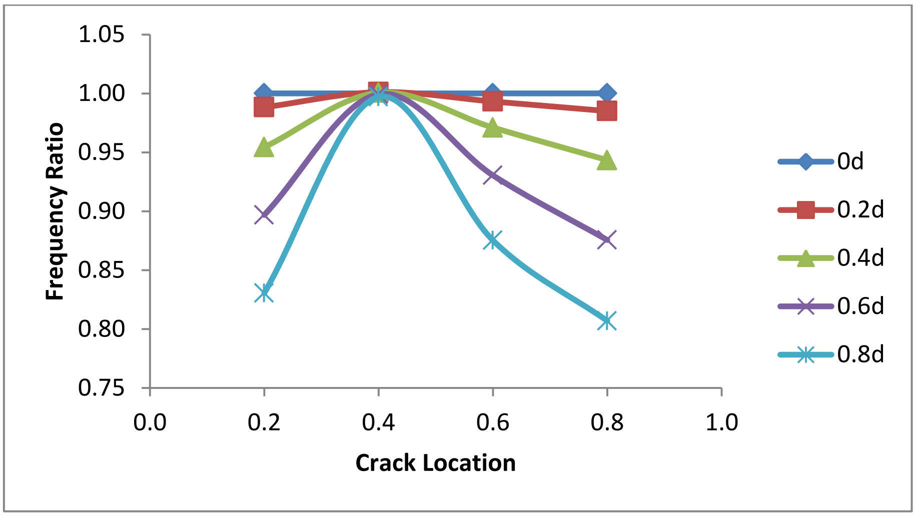


Figure 5.19: Variation of frequency ratio of 4th frequency with change in crack location for different crack depths under free vibration with a compressive load 0.2P.

For free vibration analysis of stepped beam under 0.2P load condition the frequency ratio of 4th frequency ratio is constant for 0.2d and for 0.4d the changes are slight, for 0.6d increases from 0.2L to 0.4L reaches 1 and then decrease linearly up to 0.8L and for 0.8d the changes are similar to 0.6d.

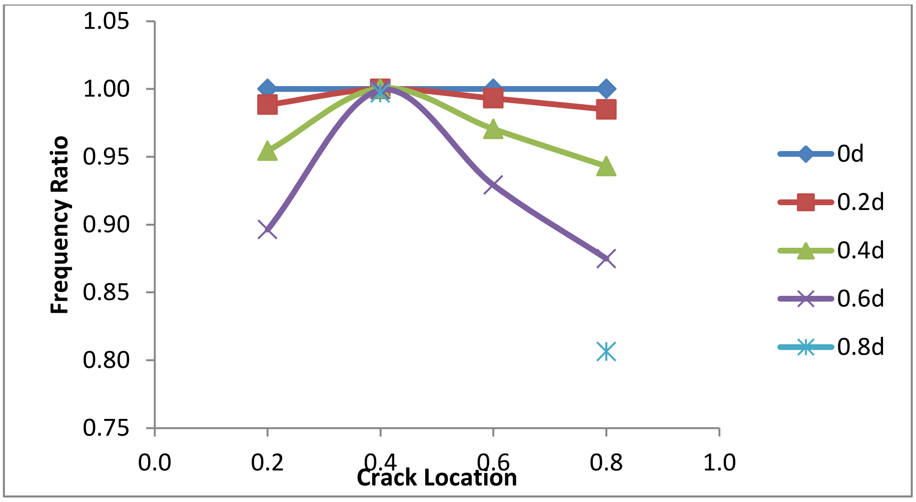


Figure 5.20: Variation of frequency ratio of 4th frequency with change in crack location for different crack depths under free vibration with a compressive load 0.4P.

For free vibration analysis of stepped beam under 0.4P load condition the frequency ratio of 4th frequency ratio is constant for 0.2d and for 0.4d the changes are slight, for 0.6d increases from 0.2L to 0.4L reaches 1 and then decrease linearly up to 0.8L and for 0.8d the column buckles except for 0.8L.

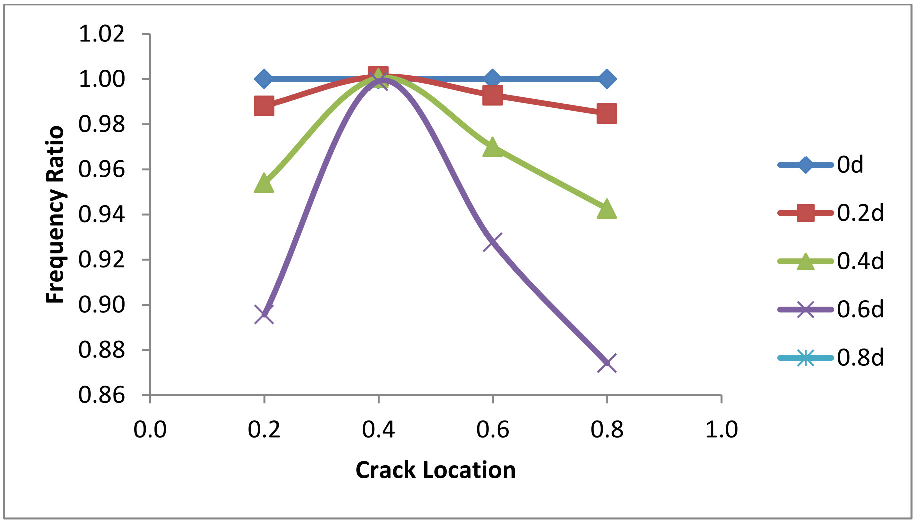


Figure 5.21: Variation of frequency ratio of 4th frequency with change in crack location for different crack depths under free vibration with a compressive load 0.6P.

For free vibration analysis of stepped beam under 0.6P load condition the frequency ratio of 4th frequency ratio is constant for 0.2d and for 0.4d the changes are slight, for 0.6d increases from 0.2L to 0.4L reaches 1 and then decrease linearly up to 0.8L and for 0.8d the column buckles.

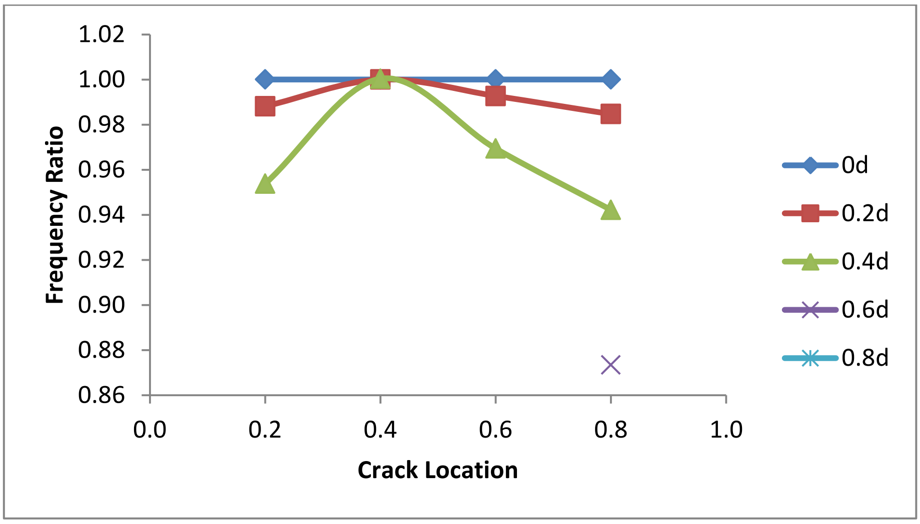


Figure 5.22: Variation of frequency ratio of 4th frequency with change in crack location for different crack depths under free vibration with a compressive load 0.8P.

For free vibration analysis of stepped beam under 0.6P load condition the frequency ratio of 4th frequency ratio is constant for 0.2d and for 0.4d the changes are slight, for 0.6 crack depth column buckle's except for 0.8L location and for 0.8d the column buckles.

5.4 Observations for decrease in percentage of frequency with increase in compressive load.

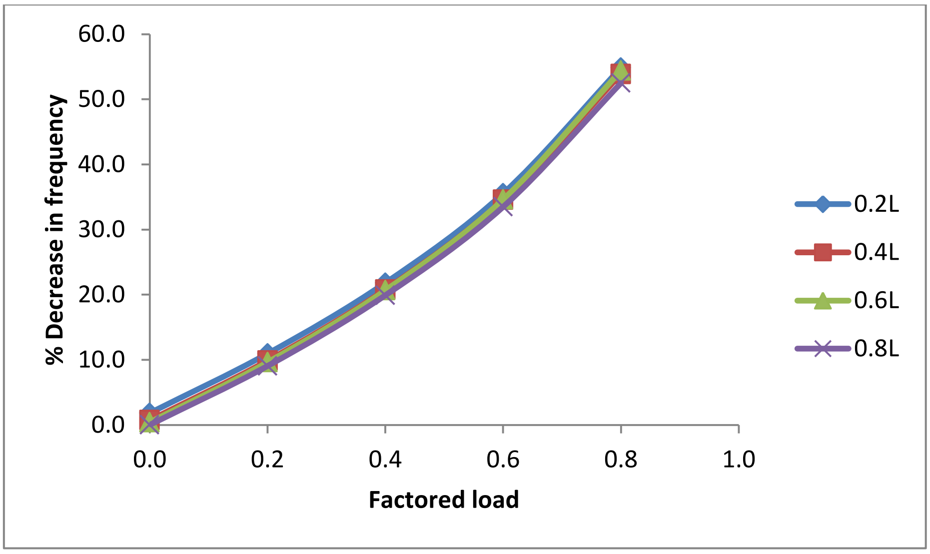


Figure 5.23: A graph between different loading percentages and % decrease in frequency for different crack locations of ω_1 and crack depth 0.2d for fixed-free boundary condition

At a crack location 0.2L and for crack depth of 0.2d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 1^{st} frequency is 1.84% at 20% of buckling load the decrease is 10.96%, at 0.4 Pcr the decrease is 21.81% and the percentage in decrease of frequency increase to 35.54% at 0.6P and it decreases by 54.84% at 0.8P.

At a crack location 0.12m and for crack depth of 0.2d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 1^{st} frequency is 0.74% at no load case and as load increase to 0.2P the decrease in frequency increase to 10.96% and with further increase in load to 0.4P the decrease in frequency becomes 21.81% and the decrease in frequency becomes to 35.54% at a load of 0.6P and it becomes 54.84% at 0.8P.

With a depth of crack 0.2d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 1^{st} frequency is 0.42% for no load case and the reduction in frequency becomes 9.63% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 20.67% and the decrease in frequency is further increased to 34.61% for a load of 0.6P and it becomes 54.39% for a load of 0.8P.

The decrease in % of 1st frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.2d is 0.032% at no load case and it is decreased by 9.06% at 0.2P and the decrease in % of frequency at 0.4P is 19.91% and it increases to 33.49% at 0.6P and at 0.8P the decrease in frequency becomes 52.48%.

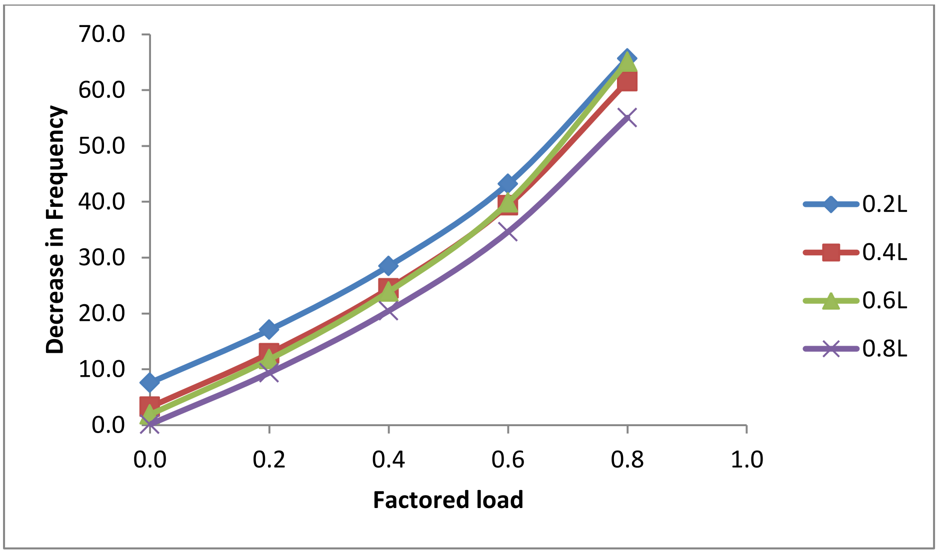


Figure 5.24: A graph between different loading percentages and % decrease in frequency for different crack locations of ω_1 and crack depth 0.4d for fixed-free boundary condition

At a crack location 0.2L and for crack depth of 0.4d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 1st frequency is 7.6% at 20% of buckling load the decrease is 17.06%, at 0.4 Pcr the decrease is 28.481% and the percentage in decrease of frequency increase to 43.24% at 0.6P and it decreases by 65.64% at 0.8P.

At a crack location 0.12m and for crack depth of 0.4d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 1st frequency is 3.26% at no load case and as load increase to 0.2P the decrease in frequency increase to 12.85% and with further increase in load to 0.4P the decrease in frequency becomes 24.46% and the decrease in frequency becomes to 39.38% at a load of 0.6P and it becomes 61.57% at 0.8P.

With a depth of crack 0.4d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 1st frequency is 1.9% for no load case and the reduction in frequency becomes 11.8% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 23.9% and the decrease in frequency is further increased to 39.86% for a load of 0.6P and it becomes 65.1% for a load of 0.8P.

The decrease in % of 1st frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.4d is 0.15% at no load case and it is decreased by 9.38% at 0.2P and the decrease in % of frequency at 0.4P is 20.4% and it increases to 34.649% at 0.6P and at 0.8P the decrease in frequency becomes 55.08%.

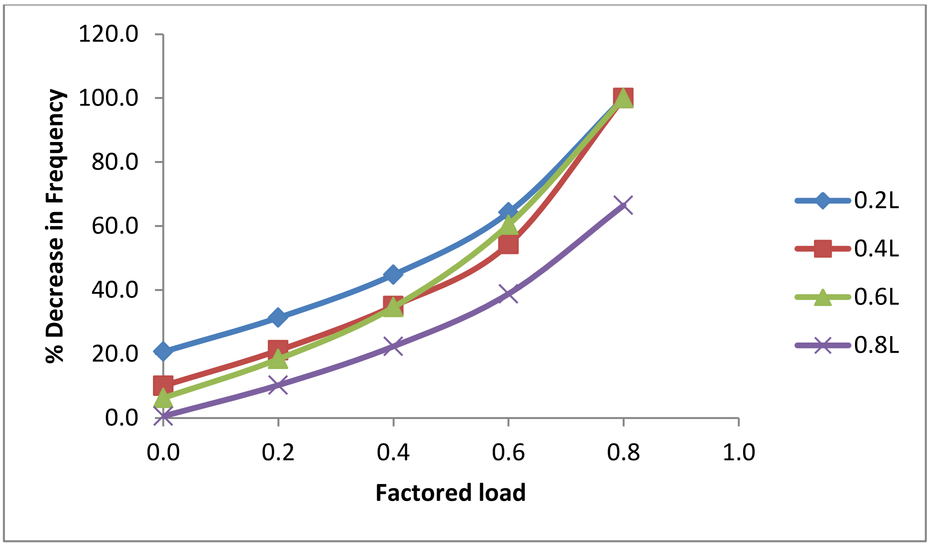


Figure 5.25: A graph between different loading percentages and % decrease in frequency for different crack locations of ω_1 and crack depth 0.6d for fixed-free boundary condition

At a crack location 0.2L and for crack depth of 0.6d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 1st frequency is 20.67% at 20% of buckling load the decrease is 31.34%, at 0.4 Pcr the decrease is 44.75% and the percentage in decrease of frequency increase to 64.23% at 0.6P and it decreases by 100% or fails at 0.8P.

At a crack location 0.12m and for crack depth of 0.6d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 1st frequency is 10.08% at no load case and as load increase to 0.2P the decrease in frequency increase to 21.1% and with further increase in load to 0.4P the decrease in frequency becomes 34.94% and the decrease in frequency becomes to 54.31% at a load of 0.6P and it becomes 100% or fails at 0.8P.

With a depth of crack 0.6d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 1st frequency is 6.23% for no load case and the reduction in frequency becomes 18.4% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 34.6% and the decrease in frequency is further increased to 60.37% for a load of 0.6P and it becomes 100% for a load of 0.8P.

The decrease in % of 1st frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.6d is 0.53% at no load case and it is decreased by 10.25% at 0.2P and the decrease in % of frequency at 0.4P is 22.37% and it increases to 38.649% at 0.6P and at 0.8P the decrease in frequency becomes 66.43%.

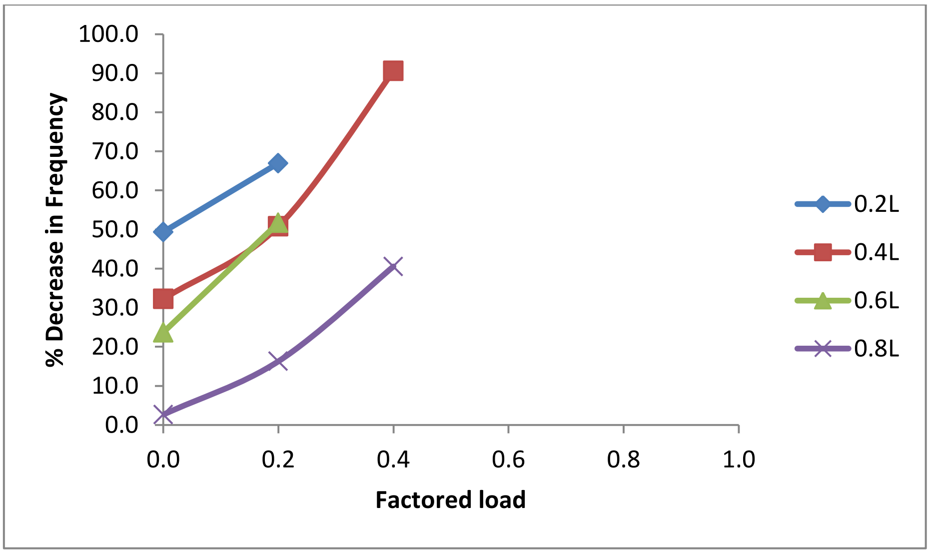


Figure 5.26: A graph between different loading percentages and % decrease in frequency for different crack locations of ω_1 and crack depth 0.8d for fixed-free boundary condition

At a crack location 0.2L and for crack depth of 0.8d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 1st frequency is 49.39% at 20% of buckling load the decrease is 66.97%, at 0.4 Pcr the decrease is 100% and the percentage in decrease of frequency increase to 100% at 0.6P and it decreases by 100% or fails at 0.8P.

At a crack location 0.12m and for crack depth of 0.8d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 1st frequency is 32.23% at no load case and as load increase to 0.2P the decrease in frequency increase to 50.84% and with further increase in load to 0.4P the decrease in frequency becomes 90.55% and the decrease in frequency becomes to 100% at a load of 0.6P and it becomes 100% or fails at 0.8P.

With a depth of crack 0.8d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 1st frequency is 23.7% for no load case and the reduction in frequency becomes 51.72% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 100% and the decrease in frequency is further increased to 100% for a load of 0.6P and it becomes 100% for a load of 0.8P.

The decrease in % of 1st frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.8d is 2.65% at no load case and it is decreased by 16.28% at 0.2P and the decrease in % of frequency at 0.4P is 40.55% and it increases to 100% at 0.6P and at 0.8P the decrease in frequency becomes 100%.

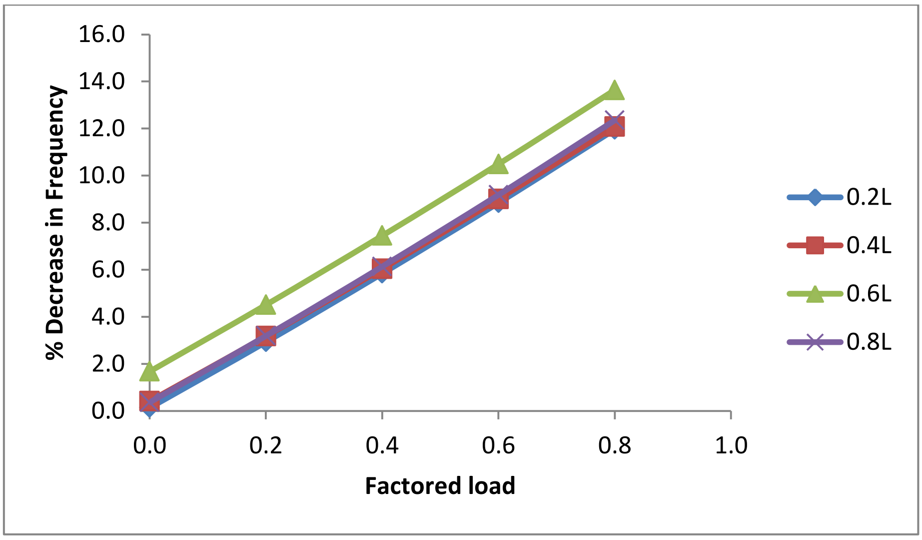


Figure 5.27: A graph between different loading percentages and % decrease in frequency for different crack locations of ω_2 and crack depth 0.2d for fixed-free boundary condition

At a crack location 0.2L and for crack depth of 0.2d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 2nd frequency is 0.135% at 20% of buckling load the decrease is 2.93%, at 0.4 Pcr the decrease is 5.84% and the percentage in decrease of frequency increase to 8.84% at 0.6P and it decreases by 11.96% at 0.8P.

At a crack location 0.12m and for crack depth of 0.2d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 2nd frequency is 0.413% at no load case and as load increase to 0.2P the decrease in frequency increase to 3.174% and with further increase in load to 0.4P the decrease in frequency becomes 6.04% and the decrease in frequency becomes to 9% at a load of 0.6P and it becomes 12.08% at 0.8P.

With a depth of crack 0.2d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 2nd frequency is 1.68% for no load case and the reduction in frequency becomes 4.5% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 7.44% and the decrease in frequency is further increased to 10.5% for a load of 0.6P and it becomes 13.64% for a load of 0.8P.

The decrease in % of 2nd frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.2d is 0.364% at no load case and it is decreased by 3.2% at 0.2P and the decrease in % of frequency at 0.4P is 6.13% and it increases to 9.2% at 0.6P and at 0.8P the decrease in frequency becomes 12.35%.

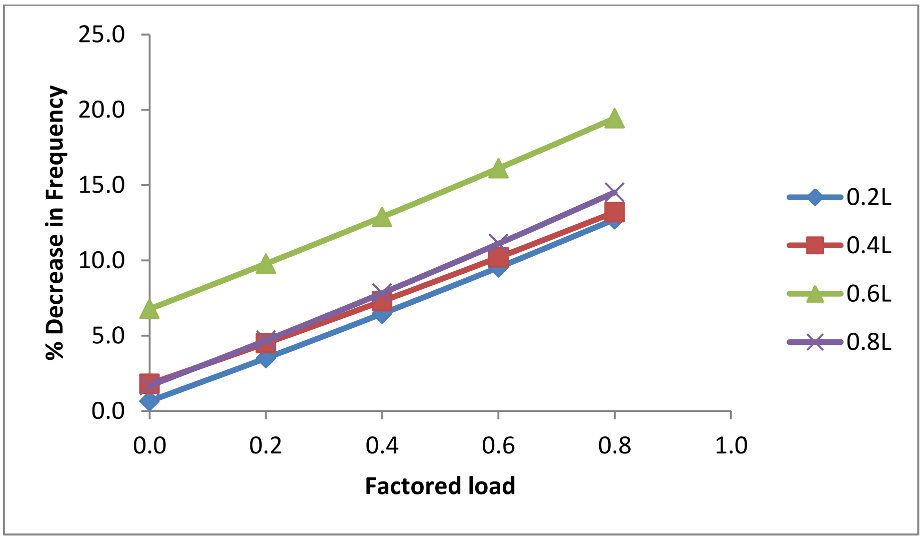


Figure 5.28: A graph between different loading percentages and % decrease in frequency for different crack locations of ω_2 and crack depth 0.4d for fixed-free boundary condition

At a crack location 0.2L and for crack depth of 0.4d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 2nd frequency is 0.65% at 20% of buckling load the decrease is 3.5%, at 0.4 Pcr the decrease is 6.45% and the percentage in decrease of frequency increase to 9.52% at 0.6P and it decreases by 12.72% at 0.8P.

At a crack location 0.12m and for crack depth of 0.4d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 2nd frequency is 1.78% at no load case and as load increase to 0.2P the decrease in frequency increase to 4.5% and with further increase in load to 0.4P the decrease in frequency becomes 7.3% and the decrease in frequency becomes to 10.2% at a load of 0.6P and it becomes 13.2% at 0.8P.
With a depth of crack 0.4d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 2nd frequency is 1.68% for no load case and the reduction in frequency becomes 4.5% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 7.44% and the decrease in frequency is further increased to 10.5% for a load of 0.6P and it becomes 13.64% for a load of 0.8P.

The decrease in % of 2nd frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.4d is 1.67% at no load case and it is decreased by 4.68% at 0.2P and the decrease in % of frequency at 0.4P is 7.82% and it increases to 11.1% at 0.6P and at 0.8P the decrease in frequency becomes 14.51%.

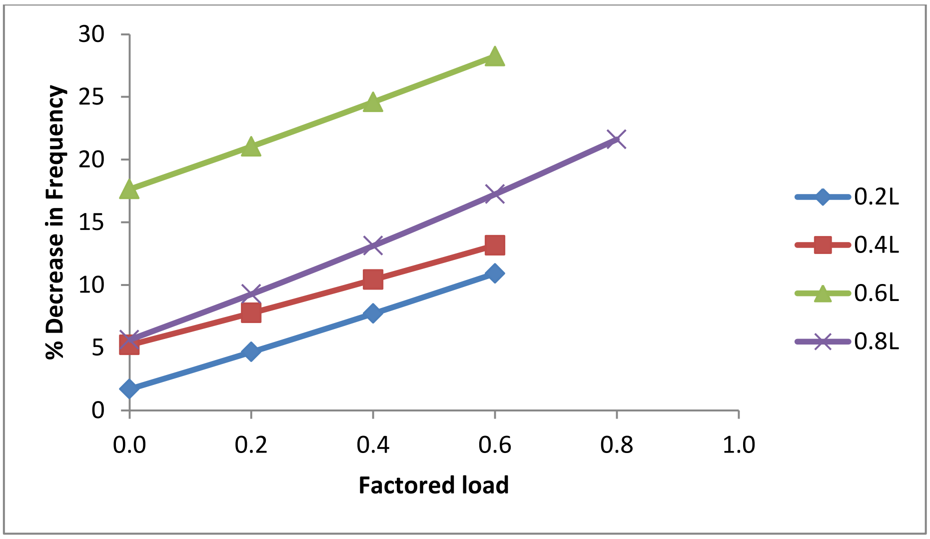


Figure 5.29: A graph between different loading percentages and % decrease in frequency for different crack locations of ω_2 and crack depth 0.6d for fixed-free boundary condition.

At a crack location 0.2L and for crack depth of 0.6d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 2^{nd} frequency is 1.68% at 20% of buckling load the decrease is 4.65%, at 0.4 Pcr the decrease is 7.71% and the percentage in decrease of frequency increase to 10.92% at 0.6P and it decreases by 100% at 0.8P. Because due to lateral load, column buckles at the point nearer to fixed end due to greater crack depth where the effect of the crack is higher.

At a crack location 0.12m and for crack depth of 0.6d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 2^{nd} frequency is 5.21% at no load case and as load increase to 0.2P the decrease in frequency increase to 7.76% and with further increase in load to 0.4P the decrease in frequency becomes 10.42% and the decrease in frequency becomes to 13.16% at a load of 0.6P and it becomes 100% at 0.8P. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

With a depth of crack 0.6d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 2^{nd} frequency is 17.64% for no load case and the reduction in frequency becomes 21.05% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 24.59% and the decrease in frequency is further increased to 28.24% for a load of 0.6P and it becomes 100% for a load of 0.8P. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

The decrease in % of 2^{nd} frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.6d is 5.67% at no load case and it is decreased by 9.26% at 0.2P and the decrease in % of frequency at 0.4P is 13.12% and it increases to 17.23% at 0.6P and at 0.8P the decrease in frequency becomes 21.61%.

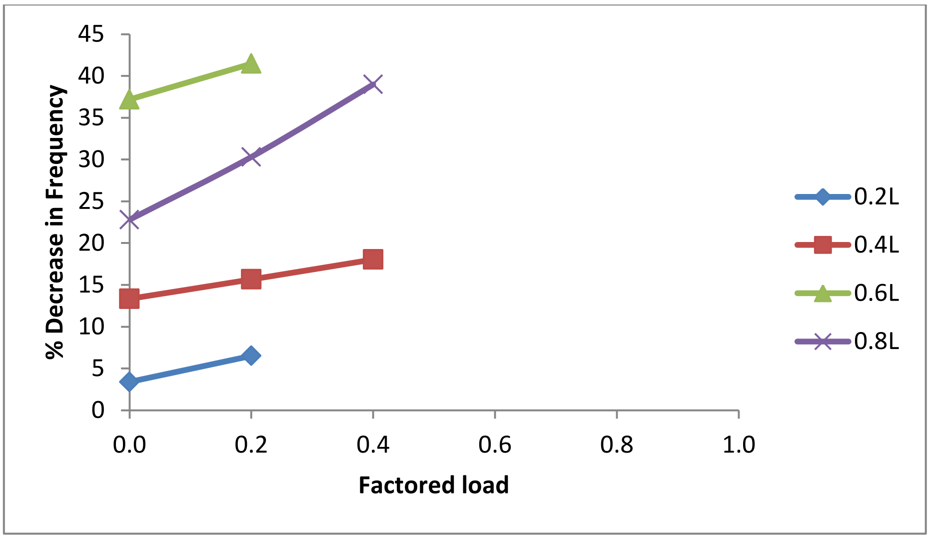


Figure 5.30: A graph between different loading percentages and % decrease in frequency for different crack locations of ω_2 and crack depth 0.8d for fixed-free boundary condition

At a crack location 0.2L and for crack depth of 0.8d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 2nd frequency is 3.4% at 20% of buckling load the decrease is 6.51%, at 0.4 Pcr the decrease is 100% and the percentage in decrease of frequency increase to 100% at 0.6P and it decreases by 100% at 0.8P. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

At a crack location 0.12m and for crack depth of 0.8d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 2nd frequency is 13.336% at no load case and as load increase to 0.2P the decrease in frequency increase to 15.66% and with further increase in load to 0.4P the decrease in frequency becomes 18.04% and the decrease in frequency becomes to 100% at a load of 0.6P and it becomes 100% at 0.8P. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

With a depth of crack 0.8d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 2nd frequency is 37.12% for no load case and the reduction in frequency becomes 41.46% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 100% and the decrease in frequency is further increased to 100% for a load of 0.6P and it becomes 100% for a load of 0.8P Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

The decrease in % of 2^{nd} frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.8d is 22.8% at no load case and it is decreased by 30.31% at 0.2P and the decrease in % of frequency at 0.4P is 38.98% and it increases to 100% at 0.6P and at 0.8P the decrease in frequency becomes 100%. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

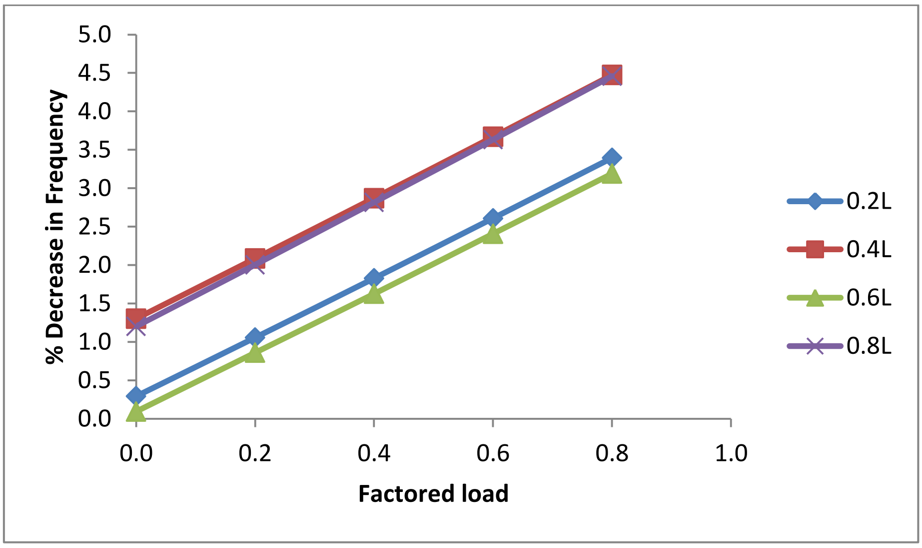


Figure 5.31: A graph between different loading percentages and % decrease in frequency for different crack locations of ω_3 and crack depth 0.2d for fixed-free boundary condition

At a crack location 0.2L and for crack depth of 0.2d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 3^{rd} frequency is 0.29% at 20% of buckling load the decrease is 1.05%, at 0.4 Pcr the decrease is 1.83% and the percentage in decrease of frequency increase to 2.61% at 0.6P and it decreases by 3.39% at 0.8P.

At a crack location 0.12m and for crack depth of 0.2d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 3^{rd} frequency is 1.3% at no load case and as load increase to 0.2P the decrease in frequency increase to 2.01% and with further increase in load to 0.4P the decrease in frequency becomes 2.87% and the decrease in frequency becomes to 3.67% at a load of 0.6P and it becomes 4.47% at 0.8P.

With a depth of crack 0.2d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 3^{rd} frequency is 0.09% for no load case and the reduction in frequency becomes 0.86% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 1.62% and the decrease in frequency is further increased to 2.41% for a load of 0.6P and it becomes 3.2% for a load of 0.8P.

The decrease in % of 3rd frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.2d is 1.2% at no load case and it is decreased by 2% at 0.2P and the decrease in % of frequency at 0.4P is 2.81% and it increases to 3.63% at 0.6P and at 0.8P the decrease in frequency becomes 4.46%.

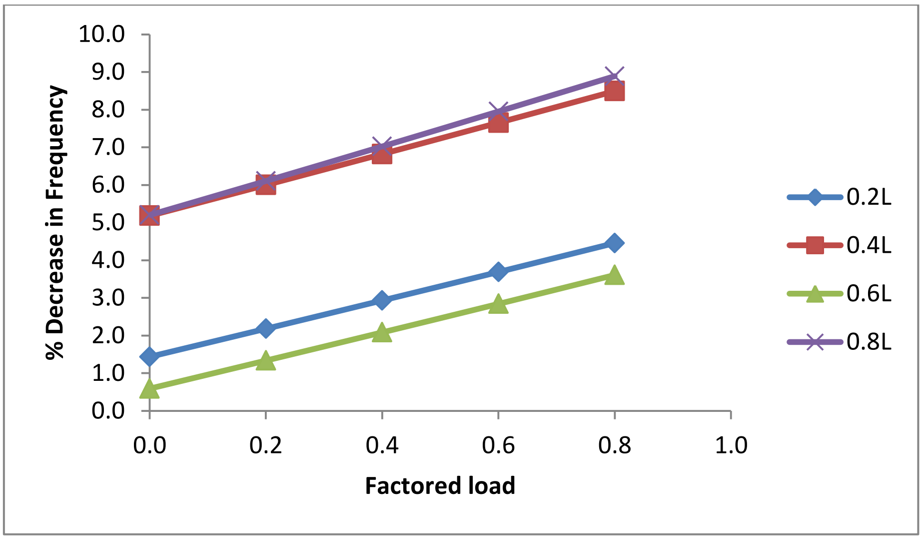


Figure 5.32: A graph between different loading percentages and % decrease in frequency for different crack locations of ω_3 and crack depth 0.4d for fixed-free boundary condition

At a crack location 0.2L and for crack depth of 0.4d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 3rd frequency is 1.43% at 20% of buckling load the decrease is 2.1%, at 0.4 Pcr the decrease is 2.93% and the percentage in decrease of frequency increase to 3.67% at 0.6P and it decreases by 4.45% at 0.8P.

At a crack location 0.12m and for crack depth of 0.4d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 3rd frequency is 5.2% at no load case and as load increase to 0.2P the decrease in frequency increase to 6% and with further increase in load to 0.4P the decrease in frequency becomes 6.82% and the decrease in frequency becomes to 7.67% at a load of 0.6P and it becomes 8.49% at 0.8P.

With a depth of crack 0.4d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 3rd frequency is 0.6% for no load case and the reduction in frequency becomes 1.33% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 2.1% and the decrease in frequency is further increased to 2.84% for a load of 0.6P and it becomes 3.6% for a load of 0.8P.

The decrease in % of 3rd frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.4d is 5.2% at no load case and it is decreased by 2% at 0.2P and the decrease in 6.1% of frequency at 0.4P is 7% and it increases to 7.95% at 0.6P and at 0.8P the decrease in frequency becomes 8.88%.

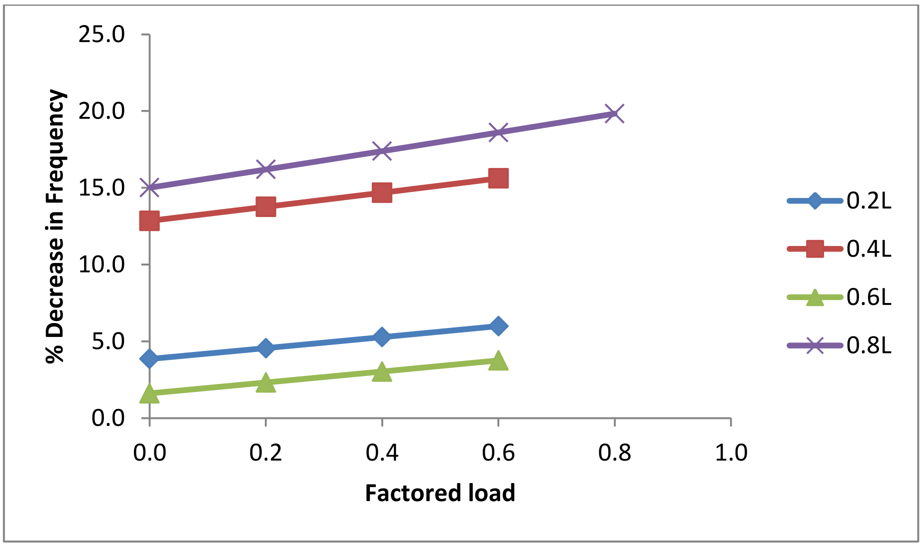


Figure 5.33: A graph between different loading percentages and % decrease in frequency for different crack locations of ω_3 and crack depth 0.6d for fixed-free boundary condition

At a crack location 0.2L and for crack depth of 0.6d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 3rd frequency is 3.84% at 20% of buckling load the decrease is 4.55%, at 0.4 Pcr the decrease is 5.24% and the percentage in decrease of frequency increase to 5.98% at 0.6P and it decreases by 100% at 0.8P. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

At a crack location 0.12m and for crack depth of 0.6d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 3rd frequency is 12.84% at no load case and as load increase to 0.2P the decrease in frequency increase to 13.76% and with further increase in load to 0.4P the decrease in frequency becomes 14.679% and the decrease in frequency becomes to 15.612% at a load of 0.6P and it becomes 100% at 0.8P. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

With a depth of crack 0.6d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 3rd frequency is 1.61% for no load case and the reduction in frequency becomes 2.31% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 3.03% and the decrease in frequency is further increased to 3.75% for a load of 0.6P and it becomes 100% for a load of 0.8P. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

The decrease in % of 3rd frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.6d is 15% at no load case and it is decreased by 16.2% at 0.2P and the decrease in 17.38% of frequency at 0.4P is 18.6% and it increases to 19.82% at 0.6P and at 0.8P the decrease in frequency becomes 8.88%.

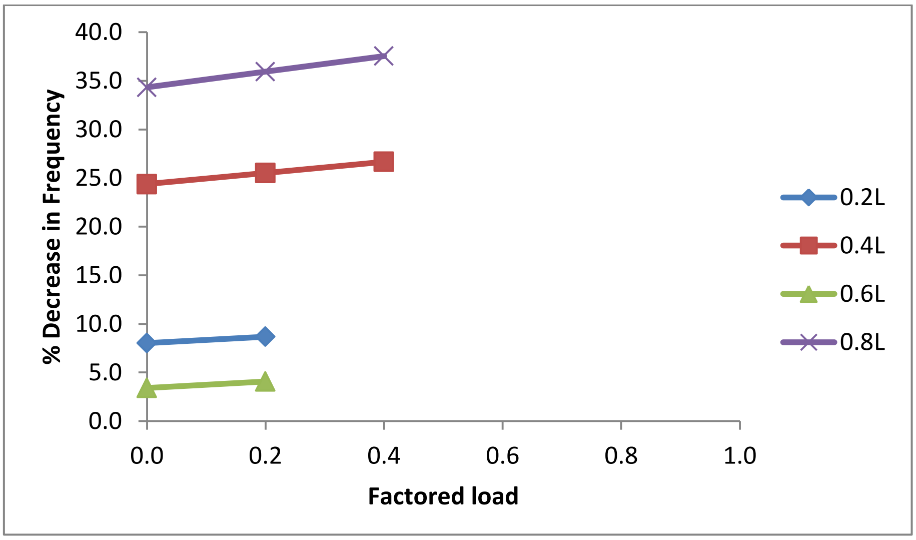


Figure 5.34: A graph between different loading percentages and % decrease in frequency for different crack locations of ω_3 and crack depth 0.8d for fixed-free boundary condition

At a crack location 0.2L and for crack depth of 0.8d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 3rd frequency is 8.02% at 20% of buckling load the decrease is 8.68%, at 0.4 Pcr the decrease is 100% and the percentage in decrease of frequency increase to 100% at 0.6P and it decreases by 100% at 0.8P. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

At a crack location 0.12m and for crack depth of 0.8d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 3rd frequency is 24.38% at no load case and as load increase to 0.2P the decrease in frequency increase to 25.51% and with further increase in load to 0.4P the decrease in frequency becomes 26.67% and the decrease in frequency becomes to 100% at a load of 0.6P and it becomes 100% at 0.8P. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

With a depth of crack 0.8d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 3rd frequency is 3.41% for no load case and the reduction in frequency becomes 4.1% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 100% and the decrease in frequency is further increased to 100% for a load of 0.6P and it becomes 100% for a load of 0.8P. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

The decrease in % of 3rd frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.8d is 34.33% at no load case and it is decreased by 35.94% at 0.2P and the decrease in 37.53% of frequency at 0.4P and it increases to 100% at 0.6P and at 0.8P the decrease in frequency becomes 100%. Because column buckles at the point nearer to fixed end due to greater crack depth, where the effect of the crack is higher.

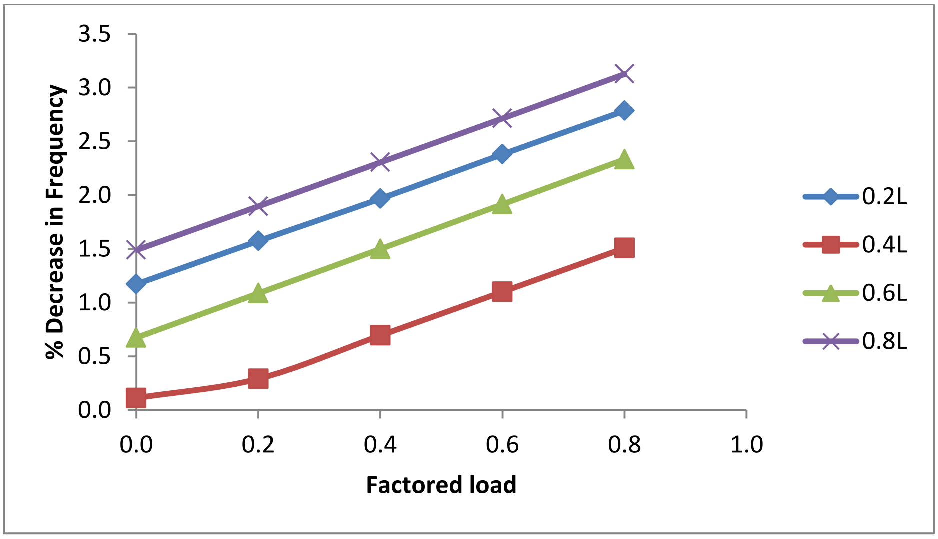


Figure 5.35: A graph between different loading percentages and % decrease in frequency for different crack locations of ω_4 and crack depth 0.2d for fixed-free boundary condition

At a crack location 0.2L and for crack depth of 0.2d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 4th frequency is 1.17% at 20% of buckling load the decrease is 1.57%, at 0.4 Pcr the decrease is 1.96% and the percentage in decrease of frequency increase to 2.38% at 0.6P and it decreases by 2.78% at 0.8P.

At a crack location 0.12m and for crack depth of 0.2d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 4th frequency is 0.122% at no load case and as load increase to 0.2P the decrease in frequency increase to 0.3% and with further increase in load to 0.4P the decrease in frequency becomes 0.7% and the decrease in frequency becomes 1.1% at a load of 0.6P and it becomes 1.5% at 0.8P.

With a depth of crack 0.2d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 4th frequency is 0.67% for no load case and the reduction in frequency becomes 1.08% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 1.5% and the decrease in frequency is further increased to 1.91% for a load of 0.6P and it becomes 2.33% for a load of 0.8P.

The decrease in % of 4th frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.2d is 1.49% at no load case and it is decreased by 1.9% at 0.2P and the decrease in 2.3% of frequency at 0.4P and it increases to 2.71% at 0.6P and at 0.8P the decrease in frequency becomes 3.12%.

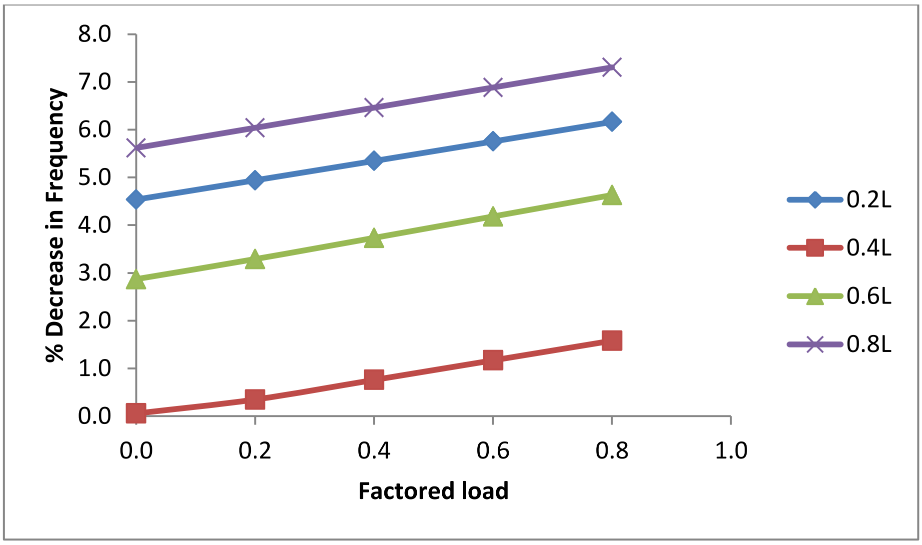


Figure 5.36: A graph between different loading percentages and % decrease in frequency for different crack locations of ω_4 and crack depth 0.4d for fixed-free boundary condition

At a crack location 0.2L and for crack depth of 0.4d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 4th frequency is 4.53% at 20% of buckling load the decrease is 4.94%, at 0.4 Pcr the decrease is 5.34% and the percentage in decrease of frequency increase to 5.75% at 0.6P and it decreases by 6.16% at 0.8P.

At a crack location 0.12m and for crack depth of 0.4d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 4th frequency is 0.056% at no load case and as load increase to 0.2P the decrease in frequency increase to 0.349% and with further increase in load to 0.4P the decrease in frequency becomes 0.76% and the decrease in frequency becomes 1.17% at a load of 0.6P and it becomes 1.58% at 0.8P.

With a depth of crack 0.4d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 4th frequency is 2.87% for no load case and the reduction in frequency becomes 3.3% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 3.73% and the decrease in frequency is further increased to 4.2% for a load of 0.6P and it becomes 4.63% for a load of 0.8P.

The decrease in % of 4th frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.4d is 5.61% at no load case and it is decreased by

6.04% at 0.2P and the decrease in 6.46% of frequency at 0.4P and it increases to 6.88% at 0.6P and at 0.8P the decrease in frequency becomes 7.3%.

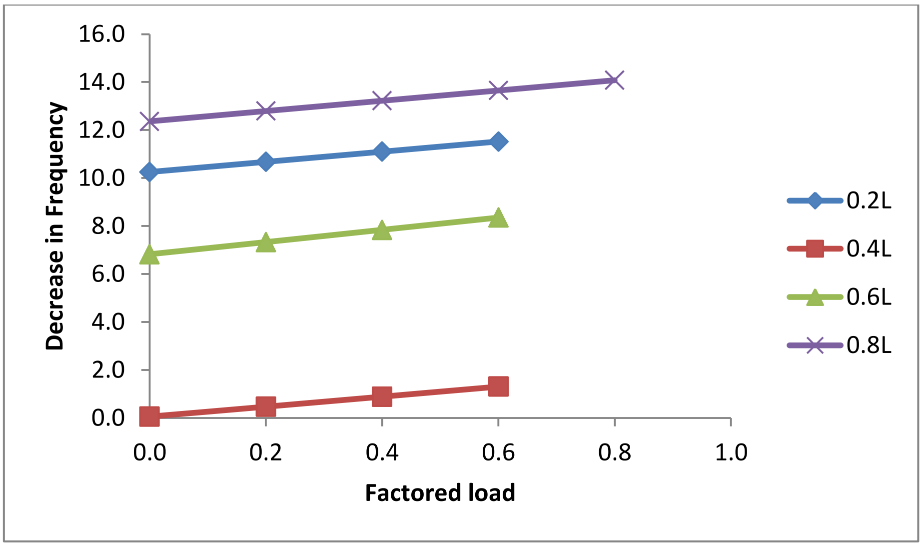


Figure 5.37: A graph between different loading percentages and % decrease in frequency for different crack locations of ω_4 and crack depth 0.6d for fixed-free boundary condition

At a crack location 0.2L and for crack depth of 0.6d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 4^{th} frequency is 10.25% at 20% of buckling load the decrease is 10.674%, at 0.4 Pcr the decrease is 11.1% and the percentage in decrease of frequency increase to 11.52% at 0.6P and it decreases by 100% at 0.8P. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

At a crack location 0.12m and for crack depth of 0.6d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 4^{th} frequency is 0.056% at no load case and as load increase to 0.2P the decrease in frequency increase to 0.466% and with further increase in load to 0.4P the decrease in frequency becomes 0.88% and the decrease in frequency becomes 1.3% at a load of 0.6P and it becomes 100% at 0.8P. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

With a depth of crack 0.6d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 4^{th} frequency is 6.82% for no load case and the reduction in frequency becomes 7.3% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 7.84% and the decrease in frequency is further increased to 8.35% for a load of 0.6P and it becomes 100% for a load of 0.8P. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

The decrease in % of 4th frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.6d is 12.36% at no load case and it is decreased by 12.8% at 0.2P and the decrease in 13.22% of frequency at 0.4P and it increases to 13.65% at 0.6P and at 0.8P the decrease in frequency becomes 14.07%.

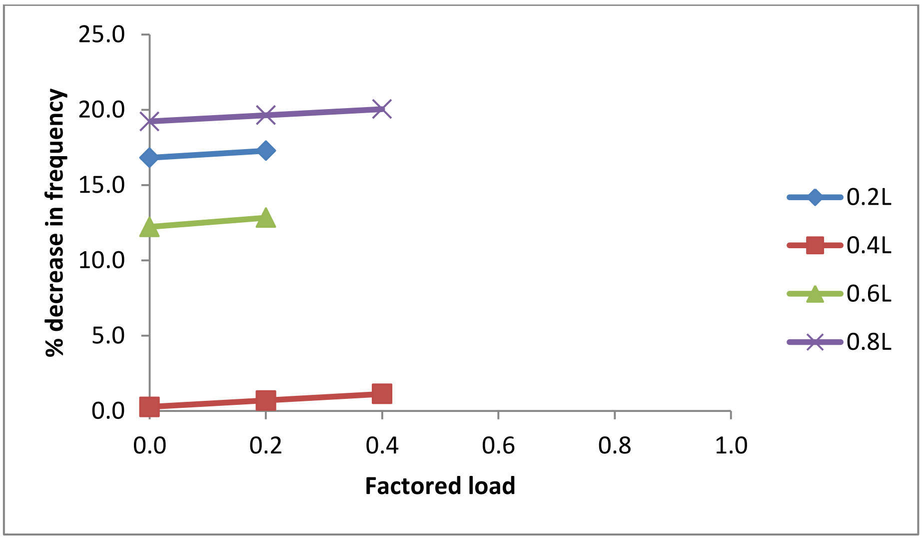


Figure 5.38: A graph between different loading percentages and % decrease in frequency for different crack locations of ω_4 and crack depth 0.8d for fixed-free boundary condition

At a crack location 0.2L and for crack depth of 0.8d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 4th frequency is 16.8% at 20% of buckling load the decrease is 17.27%, at 0.4 Pcr the decrease is 100% and the percentage in decrease of frequency increase to 100% at 0.6P and it decreases by 100% at 0.8P. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

At a crack location 0.12m and for crack depth of 0.8d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 4th frequency is 0.27% at no load case and as load increase to 0.2P the decrease in frequency increase to 0.69% and with further increase in load to 0.4P the decrease in frequency becomes 1.12% and the decrease in frequency becomes 100% at a load of 0.6P and it becomes 100% at 0.8P. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

With a depth of crack 0.8d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 4^{th} frequency is 12.22% for no load case and the reduction in frequency becomes 12.82% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 100% and the decrease in frequency is further increased to 100% for a load of 0.6P and it becomes 100% for a load of 0.8P. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

The decrease in % of 4^{th} frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.8d is 19.22% at no load case and it is decreased by 19.63% at 0.2P and the decrease in 20.03% of frequency at 0.4P and it increases to 100% at 0.6P and at 0.8P the decrease in frequency becomes 100%. Because due to lateral load, column buckles due to greater crack depth where the effect of the crack is higher.

CONCLUSIONS

From the analysis from the results and discussions following remarks and conclusions are observed.

1. The free vibration frequencies of a cracked stepped column decrease than the intact column for a crack of any depth and location in the column. However this decrease in free vibration frequencies is marginal for cracks of small depths and significant for larger crack depths. Thus the percentage of decrease in free vibration frequency increases with increase in crack depth.
2. The free vibration frequencies are more affected by the cracks present near the fixed end than free end. With the crack moving away from fixed end it loses its effect on free vibration frequencies and finally when the crack is near free end its effect is negligible.
3. When a crack in a stepped column coincides with the step of the column there is a severe drop in free vibration frequency. This is due to the fact that the section where the depth of the column decreases abruptly, there is a severe loss of stiffness of the cracked section.
4. The free-vibration frequencies under compression load decreases with increase in load than the free vibration frequencies under no load condition.
5. The load under which free vibration frequency vanishes or approaches zero can be assumed to be buckling load of the column.
6. The effect of the axial compressive load on the natural frequency can be considered linear for any value of crack depth.

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